CHAPTER 1

Introduction and Fundamental Concepts

1.1 *Introduction.*

Fluid mechanics is the study of fluids either in motion (***fluid dynamics***) or at rest (***fluid statics***) and the subsequent effects of the fluid upon the boundaries, which may be either solid surfaces or interfaces with other fluids. Both gases and liquids are classified as fluids, and the number of fluids engineering applications is enormous : breathing blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines filters, jets, and sprinklers ,to name a few. When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid. The essence of the subject of fluid flow is a judicious compromise between theory and experiment. Since fluid flow is a branch of mechanics, it satisfies a set of well documented basic laws, and thus a great deal of theoretical treatment is available. However the theory is often frustrating, because it applies mainly to idealized situations which may be invalid in practical problems.

There are two classes of fluids, ***liquids*** and ***gases***. The distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field. Since gas molecules are widely spaced with negligible cohesive forces.

1.2 *Definition of Stress.*

The force δF acting on the small element δA can be solved into two perpendicular components as given in Fig.1.1.

- The force component acting normal to the area is called normal force, is denoted by δF_n.

- The force component acting along the plane of area is called tangential force, is denoted by δF_t.

The above component forces can expressed as force per unit area, they are called as *normal stress* and *tangential stress* respectively. The tangential stress called *shear stress*.

- The normal stress is denoted by (σ) is defined as

$$\sigma = \lim_{\delta A \to 0} \frac{\delta F_n}{\delta A} \quad (1.1)$$

- The shear stress is denoted by (τ) is defined as

$$\tau = \lim_{\delta A \to 0} \frac{\delta F_t}{\delta A} \quad (1.2)$$

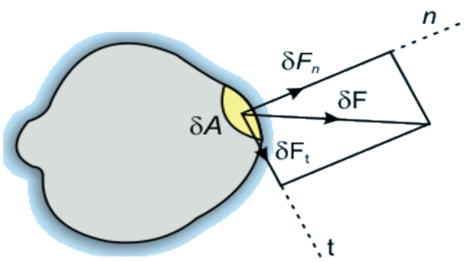


Figure1.1: Normal and tangential forces on a surface

1.3 *Definition of Fluid.*

A fluid is a substance that deforms continuously in the face of tangential or shear stress, irrespective of the magnitude of shear stress. This continuous deformation under the application of shear stress constitutes the flow. Fig.1.2 shows the shear stress on a fluid body. If a shear stress is applied at any location in a fluid, the element 011' which is initially at rest will move to 022', then to 033'. Further, it moves to 044' and continues to move in a similar fashion.

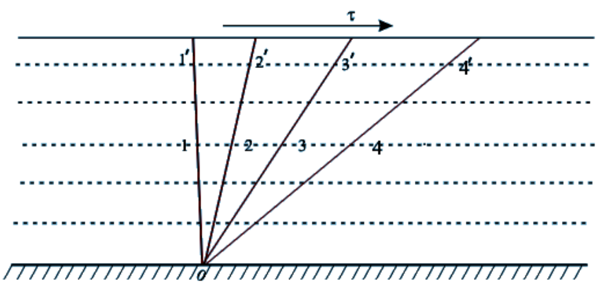


Figure1.2: Shear stress on a fluid body.

1.4 *Concept of Continuum.*

- The concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space.
- The fluid properties vary continuously from point to another. For example density at a point is defined as

$$\rho = \lim_{\Delta\forall \to 0} \left(\frac{m}{\Delta\forall}\right) \tag{1.3}$$

  Where ($\forall$) is the volume of the fluid element and (m) is the mass.
- An important note for determining the validity of continuum model is a ***molecular density.*** when (λ) is the distance between the molecules and is defined by mean free path. Which finding from statistical average distance the molecules travel between two successive collisions.
- If ***(λ)*** is very small compared with L (L characteristic length in the flow), i.e. the molecular density is very high, then the gas can be treated as continuous medium. If ***(λ)*** is large compared with L, then the gas cannot be considered continuous.
- $K_n = \lambda/L$ is the Knudsen Number, is the dimensionless number.
- If $K_n > 0.01$, the concept of continuum does not hold good.

From the range of Knudsen number, the flow is known as.

- Slip flow ($0.01 < K_n < 0.1$)
- Transition flow ($0.1 < K_n < 10$)
- Free molecule flow ($K_n > 10$)

However, for the flow regimes considered in this book, K_n is always less than 0.01 and its usual to say the fluid is continuum. In continuum approach, the fluid properties as ρ,μ,k,T, etc can be expressed as continuous function of space and time.

1.5 *Fluid Properties*.

a) Density (ρ):-

The density ***ρ*** *(rho)* may be defined as the mass per unit volume ($m/\forall$) at a standard temperature and pressure. If a fluid element enclosing a point ***P*** has a volume ***Δ∀*** and mass ***Δm*** Fig. 1.3, then density ***ρ*** at point ***P*** is written as

$$\rho = \lim_{\Delta\forall \to \Delta\forall_c} \left(\frac{m}{\Delta\forall}\right)$$

However, in a medium where continuum model is valid one can write

$$\left(\frac{m}{\Delta\forall}\right) = \left[\frac{dm}{d\forall}\right]_{\forall}$$

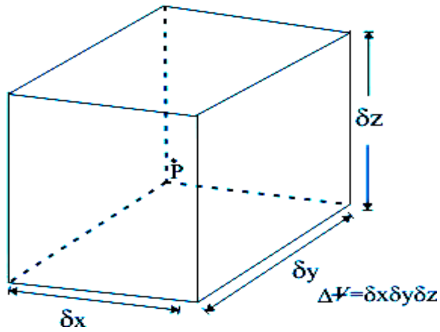


Figure 1.3: A fluid element enclosing point P

b) *Specific weight (γ):-*

The specific weight γ *(gamma)* is defined as the weight of fluid per unit volume at the standard temperature and pressure, and is given by

$$\gamma = \rho g \quad \left[\frac{N}{m^3}\right] \qquad (1.4)$$

Where g is the gravitational acceleration.

c) *Specific volume (v):-*

The specific volume ***v*** is the volume occupied by unit mass of fluid, thus

$$\upsilon = 1/\rho \quad (m^3/kg) \qquad (1.5)$$

d) *Specific gravity (S.G.):-*

The specific gravity is the ratio of density of a liquid at actual conditions to the density of pure water at ($101 kN/m^2$) and at 4C°.

$$S.G._{liquid} = \rho_{liquid}/\rho_{water} = \rho_{liquid}/1000 \qquad (1.6)$$

$$S.G._{gas} = \rho_{gas}/\rho_{air} = \rho_{gas}/1.205 \qquad (1.7)$$

e) *Temperature (T):-*

The temperature is a measure of the internal energy level of a fluid.

f) *Pressure (p):-*

The pressure is the stress at a point in a static fluid, and is the differences or gradients in pressure after drive a fluid flow especially in ducts.

1.6 *Viscosity (μ).*

A fluid is defined as a material which will continue to deform with the application of a shear force. However, different fluids deform at different rates when the same shear stress (force/area) is applied. If the force ***F*** acts over an area of contact ***A***, then the shear stress τ is defined as

$$\tau = F/A \qquad (1.8)$$

The shear strain angle $\delta\theta$ as in Fig. 1.4.a will continuously grow with time as long as the shear stress τ is maintained. The upper surface moving at speed δu larger than the lower. For any fluid water, oil and air show a linear relation between applied shear and resulting strain rate

$$\tau \propto \delta\theta / \delta t \qquad (1.9)$$

From the geometry of Fig. 1.4.a we see that

$$\tan \delta\theta = \delta u\, \delta t / \delta y \qquad (1.10)$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient.

$$\frac{d\theta}{dt} = \frac{du}{dy} \qquad (1.11)$$

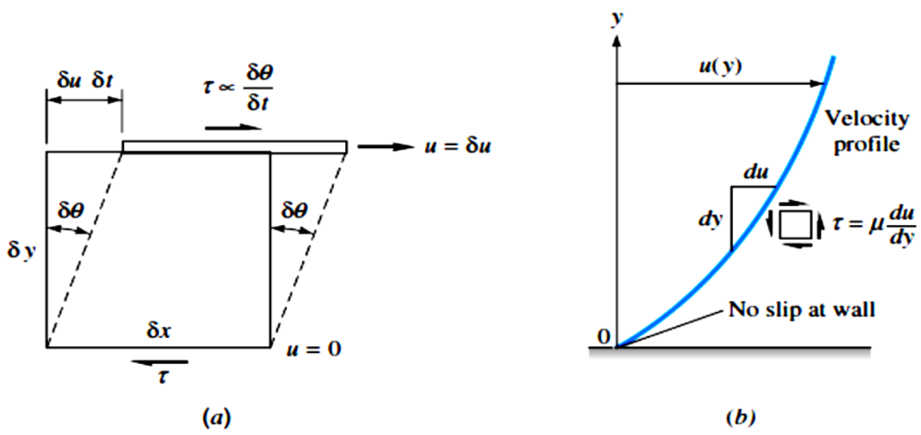


Figure1.4: **(a)** A fluid element straining at rate δθ/δt.
(b) Newtonian shear distribution in a shear layer.

Compare Eq. 1.9 with Eq. 1.11, for the common linear fluids the shear is also proportional to the velocity gradient, by using the constant of proportionality is the viscosity coefficient (μ).

$$\tau = \mu\, d\theta / dt = \mu\, du / dy$$

$$\tau = \mu\, du / dy \qquad (1.12)$$

(τ) is positive in the direction of the coordinate parallel to them. This result indicates that for common fluids such as water, oil, gasoline and air the shearing stress and rate of shearing strain (velocity gradient) can be related with Eq. 1.12. The constant of proportionality is designated by the Greek symbol μ (mu) and is called the *absolute viscosity*, *dynamic viscosity*, or *viscosity* of the fluid. Eq. 1.12 is known as Newton's law of viscosity. All fluids (water, air, oil and mercury), obey Newton's law of viscosity are known as Newtonian fluids. Other classes of fluids are known as non-Newtonian fluid as (paints, polymer solution and blood).

From above the viscosity of fluid may be defined as the property of a real fluid by virtue of which is offers resistance to shear force. Newton's law of viscosity states that the shear force to be applied for a deformation rate of (du/dy) over an area (A) is given by

$F = \mu A(du/dy)$

Or $(F/A) = \tau = \mu (du/dy) = \mu (u/y)$ (1.13)

1.7 *Causes of Viscosity.*

The causes of viscosity in a fluid are possibly attributed to two factors

- ***Intermolecular force of cohesion.***
- ***Molecular momentum transfer.***

In the flow of liquids and gases molecules are free to move from one layer to another. When the velocity in the layers are different as in viscous flow, the molecules moving from the layer at lower speed to the layer at higher speed have to be accelerated. Similarly the molecules moving from the layer at higher velocity to a layer at a lower velocity carry with them a higher value of momentum and these are to be slowed down. Thus the molecules diffusing across layers transport a net momentum introducing a shear stress between the layers. The force will be zero if both layers move at the same speed or if the fluid is at rest.

When cohesive forces exist between atoms or molecules these forces have to be overcome for relative motion between layers. A shear force is to be exerted to cause fluids to flow.

Viscous forces can be considered as the sum of these two, namely, the force due to momentum transfer and the force for overcoming cohesion. In the case of liquids, the viscous forces are due more to the breaking of cohesive forces than due to momentum transfer (as molecular velocities are low). In the case of gases viscous forces are more due to momentum transfer as distance between molecules is larger and velocities are higher.

For Newtonian fluids the coefficient of viscosity depends strongly on temperature but varies very little with pressure

- For liquids, molecular motion is less significant than the forces of cohesion, thus viscosity of liquids decrease with increase in temperature.
- For gases, molecular motion is more significant than the cohesive forces, thus viscosity of gases increase with increase in temperature.

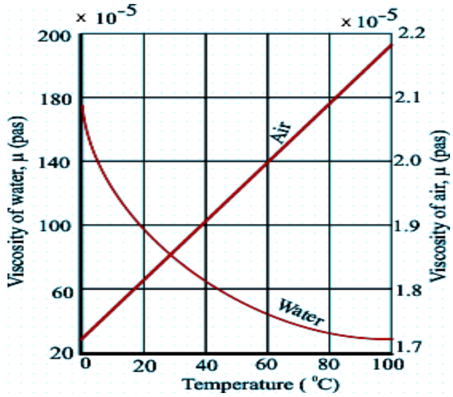


Figure 1.5: Change of water and air viscosity with temperature under 1 atm.

From above discussions about the fluid circumstances we can conclude that;

- ***Ideal Fluid*:-** Such a fluid having zero viscosity ***(μ=0)*** is called an ideal fluid and the resulting motion is called ideal fluid or inviscid flow. From this definition there is no existence of shear force.
- ***Real Fluid:-*** All fluids in reality having viscosity ***(μ > 0.0)*** are termed real fluid and their motion is known as viscous flow.
- ***Kinematic Viscosity (υ):-*** the kinematic viscosity ***υ*** (nu) is the ratio of viscosity to mass density

$$\upsilon = \mu / \rho \quad (1.14)$$

The kinematic viscosity gives the rate of momentum flux or momentum diffusivity.

1.8 *Application of Viscosity Concept.*

1.8.1 Viscous Torque and Power-Rotating Shafts.

Ex.1

Determine the power required to run a 250 mm diameter shaft at 450 rpm in journals with uniform oil thickness of 1.5 mm. Two bearings of 300 mm width are used to support the shaft. The dynamic viscosity of oil is 0.03 Pa.s.

Sol.

Shear stress on the shaft surface = $\tau = \mu\,(du/dy) = \mu\,(u/y)$

$$u = \omega * R = \pi DN/60 = \pi * 0.25 * 450/60 = 5.9 \text{ m/s}$$

since ω is the angular velocity = $2\pi N/60$

$$\tau = 0.03 * \{(5.9 - 0) / 0.0015\} = 118 \text{ N/m}^2$$

Surface area of the two bearings, $A = 2*\pi*D*L$

Force on shaft surface (F) = $\tau * A = 118 * (2 * \pi * 0.25 * 0.3) = 55.58$ N

Torque (T) = F*D/2 = 55.58*0.25/2 =6.95 N.m
Power required (P) = ω * T=2*π*N*T/60=2*π*450*6.95/60= 327.5 W

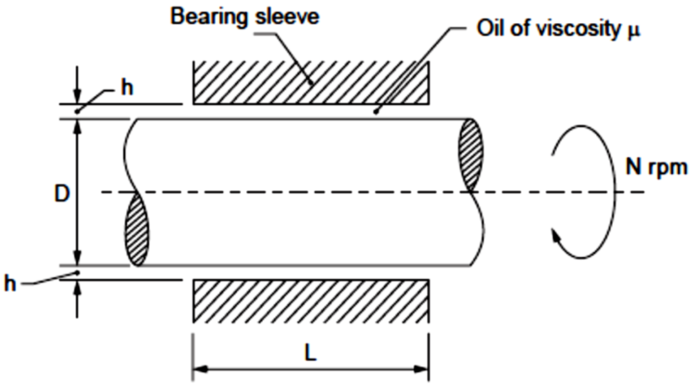


Figure 1.6: Rotating shaft in bearing

1.8.2 Viscous Torque – Disk Rotating over a Parallel Plate.

From Fig. 1.7, consider an annular strip of radius ***r*** and width ***dr.*** the force on the strip is given by

$$F=A\mu(du/dy) = A\mu(u/y)$$

As (y) is small, linear velocity variation can be assumed

$$u= \omega r= 2\pi rN/60,\ y=h,\ A=2\pi rdr$$

Torque = Force * radius

The difrential torque ***(dT)*** on the strip after substituting the above values is, $dT= 2\pi rdr\mu(2\pi rN/60h)r$

$$dT = [\mu\pi^2N/15h]r^3dr$$

Integrating the difrential torque ***(dT)*** from the center of disc to the outer edge, i.e, from r=0 to r=R will gives

$$\mathbf{T = \mu\pi^2NR^4/ 60h}$$

If the diameter is used, $R^4= D^4/16$ then,

$$\mathbf{T=\mu\pi^2ND^4/ 960h}$$

The power required, $\mathbf{P = \omega * T= 2\pi NT/60= \mu\pi^3N^2R^4/1800h}$

For an annular area like a collar the integration limits are R_o and R_i and the torque is given by

$$\mathbf{T=\mu\pi^2N(R_o^4 - R_i^4) /60\ h}$$

Power is given by $\mathbf{P= \mu\pi^3N^2 (R_o^4 - R_i^4) /1800h}$

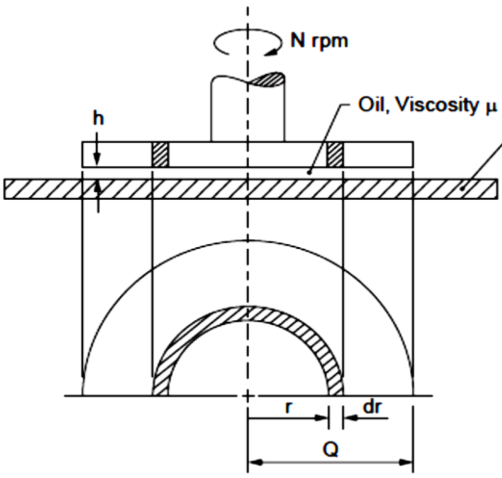


Figure 1.7: Rotating disk

Ex.2

Determine the oil film thickness between the plates of a collar bearing of 0.2 m ID and 0.3 m OD transmitting power as in below figure, if 50 W was required to overcome viscous friction while running at 700 rpm. The oil used has a viscosity of 0.003 Pa.s.

Sol.

P=2πNT/60, substituting the given values,

$$50 = 2\pi * 700 * T/60, \text{ solving for torque,}$$

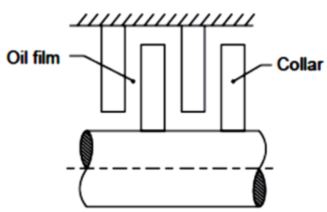
$$T = 0.682 \text{ Nm}$$

In case of an annular surface rotates over a flat surface the torque equation is

$$T = \mu\pi^2 N(R_o^4 - R_i^4)/60\,h, \text{ substituting the given values and solving for } h$$

$$0.682 = 0.003 * \pi^2 * 700 * (0.15^4 - 0.1^4)/60 * h$$

$$h = 0.000206 \text{ m} = 0.206 \text{ mm}$$



1.9 *Compressibility.*

The compressibility is the measure of its change in volume under the action of external forces. The degree of compressibility of a substance is characterized by the bulk modulus of elasticity ***E***, defined as

$$E = \lim_{\Delta \forall \to 0} \left(\frac{-\Delta p}{\Delta \forall / \forall}\right) \qquad (1.15)$$

Where

$\Delta\forall$ the change in volume.

Δp the change in pressure.

$\forall$ the initial volume.

The negative sign is to make (E) positive. For a given mass of a substance, the change in its volume and density satisfies the relation

$\Delta m = 0.0$, $\Delta(\rho\forall) = 0.0$

$\rho\Delta\forall + \forall\Delta\rho = 0.0$

$$\frac{\Delta\forall}{\forall} = -\frac{\Delta\rho}{\rho} \qquad (1.16)$$

Using the limit of (E) in Eq.(1.15), substituted in Eq.(1.16) we get

$$\left.\begin{aligned} E &= \lim_{\Delta\rho \to 0}\left(\frac{\Delta p}{\Delta\rho/\rho}\right) = \rho\frac{dp}{d\rho} \\ E &= -\frac{dp}{\Delta\forall/\forall} \end{aligned}\right\} \qquad (1.17)$$

The values of (E) for liquids are very high as compared with those of gases. Therefore the liquids are usually termed as incompressible fluids.

For example $E_{water} = 2*10^6$ kN/m^2

$E_{air} = 101$ kN/m^2

Indicates that the air is about (20000) times more compressible than water. Hence water can be treated as incompressible. For gases another characteristic parameter known as compressibility ***K***, it's the reciprocal of ***E***

$$K = \frac{1}{E} = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right) = -\frac{1}{\forall}\left(\frac{d\forall}{dp}\right) \qquad (1.18)$$

For any gaseous substance, a change in pressure is generally associated with a change in volume and a change in temperature simultaneously. A functional relationship between the pressure, volume and temperature at any equilibrium state is known as thermodynamic equation of state for the gas. For an ideal gas, the thermodynamic equation of state is given by

$$p = \rho R T \qquad (1.19)$$

R is known as the characteristic gas constant =287 J/kg.k

Distinction between an incompressible and a compressible flow from Bernoulli's equation $\boldsymbol{p + (1/2)\,\rho V^2}$ = constant, change in pressure, $\boldsymbol{\Delta p}$, in flow field is of the order $(1/2\ \rho V^2)$

$$E = \rho\, dp/d\rho \quad \text{gives} \quad \Delta\rho/\rho \cong (1/2\ \rho V^2)/E \qquad (1.20)$$

So, if $\Delta\rho/\rho$ is very small, the flow of gases can be treated as incompressible, from Laplace equation in gas flow, the velocity of sound is given by

$$i = \sqrt{\frac{E}{\rho}}$$

$$\Delta\rho/\rho \cong (1/2)V^2/i^2 \cong (1/2)\,(\boldsymbol{Ma})^2$$

Where ***Ma*** is the Mach number

If $\Delta\rho/\rho$ is maximum relative change in density of about (5%) as the criterion of ***Ma*** becomes approximately (0.33) at standard condition (i= 335.28 m/s, V_{air}=110 m/s).

Ex.3

A liquid compressed in a cylinder has a volume of 1000 cm^3 at 1MN/m^2 and a volume of 995 cm^3 at 2 MN/m^2. What is its bulk modulus of elasticity (E)?

Sol.

$$E = -\frac{\Delta p}{\Delta\forall/\forall} = -\frac{(2-1)*10^6}{(995-1000)*10^{-6}/(1000*10^{-6})} = 200MPa$$

Ex.4

If E=2.2 GPa is the bulk modulus of elasticity for water, what pressure is required to reduce a volume by 0.6 percent?

Sol.

$$E = -\frac{\Delta p}{\Delta\forall/\forall}\; 2.2*10^9 = -\frac{p_2-0}{-0.006} \qquad p_2 = 13.2 \text{ MPa}$$

1.10 *Surface Tension of Liquids.*

1.10.1 Surface Tension Phenomenon.

The phenomenon of surface tension arises due to the two kinds intermolecular forces.

I. *Cohesion Force*:- the force of attraction between the molecules of a liquid due to, they are bound to each other to remain as one assemblage of particles is known as the force of cohesion.

II. *Adhesion Force:-* The force of attraction between unlike molecules, i.e, between the molecules of different liquids or between the molecules of a liquid and those of solid body when they are in contact with each other.

A thin layer of few atomic thicknesses at the surface formed by the cohesive bond between atoms slows down and sends back the molecules reaching the surface. This cohesive bond exhibits a tensile strength for the surface layer and this is known as surface tension. Force is found necessary to stretch the surface. Surface tension may also be defined as the work per unit area (N.m / m^2) or (N/m) required creating unit surface of the liquid. The work is actually required for pulling up the molecules with lower energy from below, to form the surface. In liquids cohesion forces between molecules and the effect on solid-liquid interface are lead to surface tension. The formation of droplets is a direct effect of this phenomenon. At the interface between solid and liquid, the liquid surface being moved up or down forming a curved surface. When the adhesive forces are higher the contact surface is lifted up forming a concave surface. Oils, water etc. exhibit such behavior. These are said to be surface wetting. When the adhesive forces are lower, the contact surface lowered at the interface and convex surface results as in the case of mercury. Such liquids are called non-wetting. These are shown in Fig.1.8.

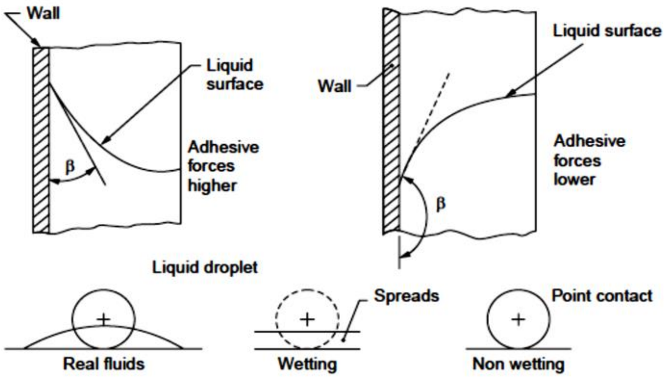


Figure 1.8: Surface tension effect at solid-liquid interface

1.10.2 Capillarity.

When a liquid is in contact with a solid, if the forces of adhesion between the molecules of the liquid and the solid are greater than the forces of cohesion among the liquid molecules themselves, the liquid molecules crowd towards the solid surface. The area of contact between the liquid and solid increases and the liquid thus wets the solid surface

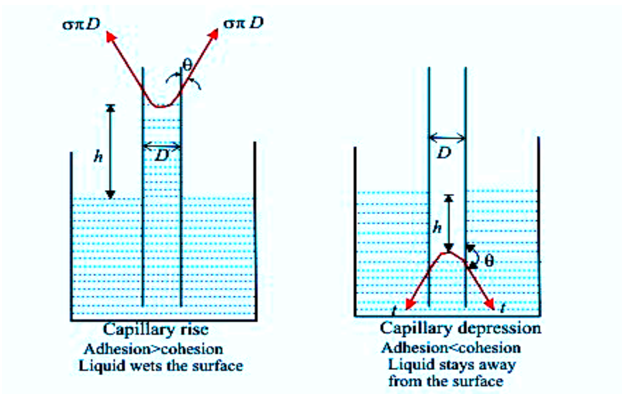


Figure 1.9: Phenomenon of Capillarity

The reverse phenomenon takes place when the force of cohesion is greater than the force of adhesion. These adhesion and cohesion properties result in the phenomenon of capillarity by which a liquid either rises or falls in a tube dipped into the liquid depending upon whether the force of adhesion is more than that of cohesion or not. Fig.1.9 shows the phenomenon of capillarity.

Ex.5

(a) Derive an expression for the change in high (h) in a circular tube of a liquid with surface tension *(σ)* and contact angle *(θ)*. As in below figure.

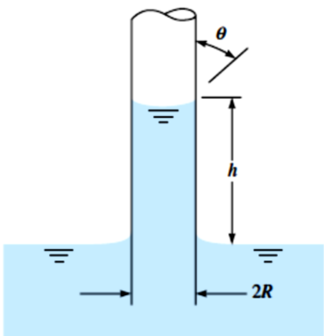
(b) Suppose that, the fluid is water having $\sigma = 0.073\ N/m$, $\theta = 0.0°$, ρ=1000 kg/m^3 and $R=1mm$, then find the capillary rise for the water-air-glass interface.

Sol. ***(a)*** The vertical component of the ring surface tension force at the interface in the tube must balance the weight of column of fluid of height *(h).*

$$2\pi R\sigma\cos\theta = \rho g\pi R^2 h$$

Solving for *h*, we have the desired result

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{4\sigma\cos\theta}{\gamma d}\ .$$



(b)

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{2(0.073)\cos 0}{1000 * 9.81 * 0.001} = 0.015\ m = 1.5\ cm$$

1.11 Dimensions and Units.

Dimensions:- is the measure by which a physical variable is expressed quantitatively.

Unit:- is a particular way of attaching a number to the quantitive dimension.

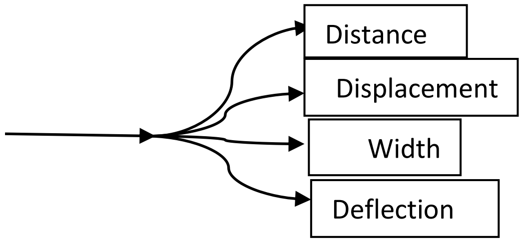
There are three widely used systems of units in the word. These are

I. British or English system (it's not in official use now in Briton)

II. Metric system.

III. SI system (System International of Unites or International System of Units).

Length: is a dimension



Centimeter
Meter
Inch

} Numerical values are the units

To standardize the metric system, a general conference of weights and measures attended in 1960 by 40 countries proposed the International system of units (SI). The British gravitational (BG) unit and SI units will be use.

In fluid mechanics there are only four primary dimensions from which all other dimensions can be derived: mass, length, time, temperature. These dimensions and their units in both systems are given in Table 1.1

Table 1.1: Primary dimensions in SI and BG system

Primary Dimension	SI Unit	BG Unit	Conversion Factor
Mass (M)	Kilogram(kg)	Slug	1 slug = 14.5939 kg
Length (L)	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time (T)	Second (s)	Second (s)	1 s = 1 s
Temperature (θ)	Kelvin (K)	Rankin (°R)	1K = 1.8 °R

The secondary dimensions, which is directly related to mass, length, time and temperature. As the force from Newton's second law

$F = m*a$

We define the Newton and pound is the dimension of force

1 Newton of force = 1N = 1 kg.m/s^2

1 pound of force = 1 lb_f = 1 slug. ft/s^2 = 4.4482 N

A list of some important secondary variables in fluid mechanics with dimensions derived as combinations of the four primary dimensions is given in Table 1.2. A more complete list of conversion factors are given in Tables 1.3&1.4.

Table 1.2 Primary dimension and conversion factor.

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88\ Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	J = N · m	ft · lbf	1 ft · lbf = 1.3558 J
Power $\{ML^2T^{-3}\}$	W = J/s	ft · lbf/s	1 ft · lbf/s = 1.3558 W
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	kg/(m · s)	slugs/(ft · s)	1 slug/(ft · s) = 47.88 kg/(m · s)
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot °R)$

Table 1.3 Conversion Factors-1.

Energy	Power
1 ft · lbf = 1.35582 J 1 Btu = 252 cal = 1055.056 J = 778.17 ft · lbf 1 kilowatt hour (kWh) = 3.6 E6 J	1 hp = 550 ft · lbf/s = 745.7 W 1 ft · lbf/s = 1.3558 W
Specific weight	**Density**
$1\ lbf/ft^3 = 157.09\ N/m^3$	$1\ slug/ft^3 = 515.38\ kg/m^3$ $1\ lbm/ft^3 = 16.0185\ kg/m^3$ $1\ g/cm^3 = 1000\ kg/m^3$
Viscosity	**Kinematic viscosity**
1 slug/(ft · s) = 47.88 kg/(m · s) 1 poise (P) = 1 g/(cm · s) = 0.1 kg/(m · s)	$1\ ft^2/h = 0.000025806\ m^2/s$ $1\ stokes\ (St) = 1\ cm^2/s = 0.0001\ m^2/s$

Temperature scale readings

$T_F = \frac{9}{5}T_C + 32$ $\quad T_C = \frac{5}{9}(T_F - 32)$ $\quad T_R = T_F + 459.69$ $\quad T_K = T_C + 273.16$

where subscripts F, C, R, and K refer to readings on the Fahrenheit, Celsius, Kelvin, and Rankine scales, respectively

Specific heat or gas constant*	Thermal conductivity*
1 ft · lbf/(slug · °R) = 0.16723 N · m/(kg · K) 1 Btu/(lb · °R) = 4186.8 J/(kg · K)	1 Btu/(h · ft · °R) = 1.7307 W/(m · K)

*Although the absolute (Kelvin) and Celsius temperature scales have different starting points, the intervals are the same size: 1 kelvin = 1 Celsius degree. The same holds true for the nonmetric absolute (Rankine) and Fahrenheit scales: 1 Rankine degree = 1 Fahrenheit degree. It is customary to express temperature differences in absolute-temperature units.

Table 1.4 Conversion Factors-2.

Length	Volume
1 ft = 12 in = 0.3048 m	1 ft^3 = 0.028317 m^3
1 mi = 5280 ft = 1609.344 m	1 U.S. gal = 231 in^3 = 0.0037854 m^3
1 nautical mile (nmi) = 6076 ft = 1852 m	1 L = 0.001 m^3 = 0.035315 ft^3
1 yd = 3 ft = 0.9144 m	1 U.S. fluid ounce = 2.9574 E-5 m^3
1 angstrom (Å) = 1.0 E-10 m	1 U.S. quart (qt) = 9.4635 E-4 m^3
Mass	**Area**
1 slug = 32.174 lbm = 14.594 kg	1 ft^2 = 0.092903 m^2
1 lbm = 0.4536 kg	1 mi^2 = 2.78784 E7 ft^2 = 2.59 E6 m^2
1 short ton = 2000 lbm = 907.185 kg	1 acre = 43,560 ft^2 = 4046.9 m^2
1 tonne = 1000 kg	1 hectare (ha) = 10,000 m^2
Velocity	**Acceleration**
1 ft/s = 0.3048 m/s	1 ft/s^2 = 0.3048 m/s^2
1 mi/h = 1.466666 ft/s = 0.44704 m/s	
1 kn = 1 nmi/h = 1.6878 ft/s = 0.5144 m/s	
Mass flow	**Volume flow**
1 slug/s = 14.594 kg/s	1 gal/min = 0.002228 ft^3/s = 0.06309 L/s
1 lbm/s = 0.4536 kg/s	1 × 10^6 gal/day = 1.5472 ft^3/s = 0.04381 m^3/s
Pressure	**Force**
1 lbf/ft^2 = 47.88 Pa	1 lbf = 4.448222 N = 16 oz
1 lbf/in^2 = 144 lbf/ft^2 = 6895 Pa	1 kgf = 2.2046 lbf = 9.80665 N
1 atm = 2116.2 lbf/ft^2 = 14.696 lbf/in^2 = 101,325 Pa	1 U.S. (short) ton = 2000 lbf
1 inHg (at 20°C) = 3375 Pa	1 dyne = 1.0 E-5 N
1 bar = 1.0 E5 Pa	1 ounce (avoirdupois) (oz) = 0.27801 N

Problems.

P1.1 Calculate the specific weight, specific volume, mass density and specific gravity of liquid having a volume of ***6m³*** and weight of ***44kN***.
[$\gamma=7.33*10^3\ N/m^3$, $\upsilon=1.337*10^{-3}\ m^3/kg$, $\rho=747.5\ kg/m^3$, S.G=0.747]

P1.2 A small village draws ***8630 ft³*** per day from its reservoir. Convert this water usage in to

a) Gallons per minute. ***[44.83 gallon/min]***

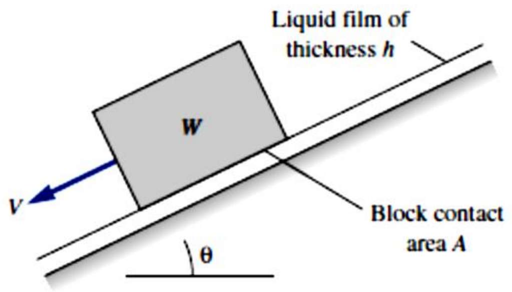
b) Liter per seconds. ***[2.82 lit/s]***

P1.3 A block of weight ***W*** slides down an inclined plane on a thin film of oil as in figure. The film contact area is ***A*** and its thickness ***h***. Assuming a linear velocity distribution in the film

a) Drive an analytic expression for the terminal velocity ***V*** of the block.

b) Find the terminal velocity if ***m=8kg***, ***$A=80cm^2$***, ***$\theta=17°$*** and the film is ***h=1mm*** thick SAE 30 oil at ***20C°***. Since ***$\mu=0.29\ kg.s/m^2$***.

[V=9.89m/s]



P1.4 The velocity distribution over a plate is given by $u=\frac{3}{2}y-\frac{1}{2}y^2$ where ***u*** is velocity ***m/s*** and ***y*** is distance from the plate boundary (m). If the viscosity of fluid is ***8 poise*** find the shear stress at the plate boundary and at ***y = 0.15 m*** from the plan (***Note 1 poise = 0.1N.s/m²***).

[τ_0=1.2 N/m², $\tau_{0.15}$=1.08 N/m²]

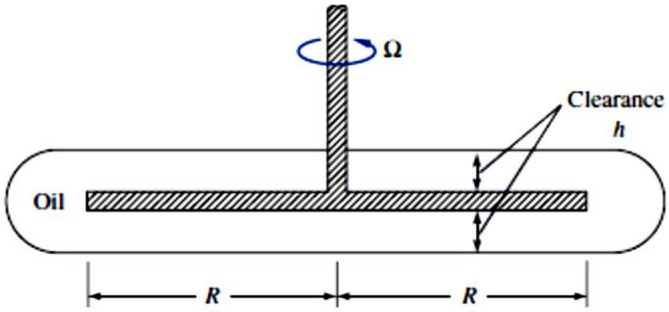
P1.5 A square metal plate ***1.5m*** side and ***1.2mm*** thick weighting ***50N*** is to be lifted through a vertical gap at ***25mm*** of infinite extent. The oil in air gap has a specific gravity of ***95*** percent and viscosity of ***2.5N.s/m²***. If the metal plate is to be lifted at a constant speed of ***0.1 m/s*** find the force and power required. ***[F=144.53 N, P=14.453 W]***

P1.6 Two large fixed parallel plane are ***240mm*** apart. The space between the surfaces is filled with oil of viscosity ***0.81N.s/m²***. A flat thin plate ***0.5m²*** area moves through the oil at a velocity of ***0.6m/s***. Calculate the drag force.

i. When the plate is equal distance from both planes. ***[F=4.05 N]***

ii. When the thin plate is at a distance of ***80mm*** from one the plate surface. ***[F=4.54 N]***

P1.7 A circular disc of radius R is slowly rotated in a liquid of large viscosity (***μ***) at a small distance ***(h)*** from a fixed surface. Drive an expression of torque (T) necessary to maintain an angular velocity (Ω) as in figure. ***[$T=\pi\mu\Omega R^4/(2h)$]***



P1.8 Determine the bulk modulus of elasticity of liquid if the pressure of the liquid is increased from ***7MN/m²*** to ***13MN/m²***. The volume of liquid decreases by ***0.15%.*** ***[E= 4 GN/m²]***

P1.9 Determine the minimum size of glass tubing that can be used to measure water level. If the capillary rise in the tube is not to exceed ***0.25mm***. Take surface tension of water in contact with air as ***0.0735N/m*** and (θ=0º) ***[d=120 mm]***

P1.10 A mercury column is used to measure the atmospheric pressure. The height of column above the mercury well surface is ***762 mm***. The tube is ***3 mm*** in diameter. The contact angle is ***140°***. Determine the true pressure in mm of mercury if surface tension is ***0.51 N/m***. The space above the column may be considered as vacuum. ***[p=765.92 mm]***

P2.11 Consider a concentric shaft fixed axially and rotated inside a cylinder.r_{sh},r_{cy} are the radius of shaft and inside radius of cylinder respectively, with total length ***L***. Let the rotational rate ***ω rad/s*** and applied torque be ***M*** .Using these parameters,

a) Derive a theoretical relation for the viscosity μ of the fluid between the shaft and cylinder. [$\boldsymbol{\mu = M(r_{cy} - r_{sh})/(2\pi\omega r_{sh}^3 L)}$]

b) For a shaft of ***8cm*** long, rotating at ***1200 rev./min***, with $\boldsymbol{r_{sh}}$ ***= 2.00 cm*** and the measured torque is ***M = 0.293 N.m***. What is the fluid viscosity? ***[μ=0.29 kg/m.s]***

CHAPTER 2

Pressure Distribution in Fluids

2.1 Introduction.

Many fluid problems do not involve motion. They concern the pressure distribution in a static fluid and its effect on solid surfaces and on floating and sub- merged bodies. When the fluid velocity is zero, denoted as the ***hydrostatic condition,*** the pressure variation is due only to the weight of the fluid. Assuming a known fluid in a given gravity field, the pressure may easily be calculated by integration. Important applications in this chapter are

I. Pressure distribution in the atmosphere and the oceans
II. The design of manometer pressure instruments.
III. Forces on submerged flat and curved surfaces.
IV. Buoyancy on a submerged body.
V. The behavior of floating bodies and its stability with the result of Archimedes principles.
VI. If the fluid is moving in ***rigid-body motion***, such as a tank of liquid which has been spinning for a long time, the pressure also can be easily calculated, because the fluid is free of shear stress. We apple this idea here to simple rigid –body accelerations.

2.2 Forces on a Fluid Elements.

Fluid Element:- is the infinitesimal region of the fluid continuum in isolation from its surroundings.

Types of forces on fluid elements:-

a) **Body Force**: it's the force which distributed over the entire mass or volume of the element, as the gravitational force, Electromagnetic force fields.
b) **Surface Force**: is the forces exerted on the fluid element by its surroundings through direct contact at the surface.

Surface force has two components

I. Normal Force: along the normal to the area.

II. Shear Force: along the plane of the area.

When the $\left[\lim_{\delta A \to 0} \frac{\delta F}{\delta A} \right] \longrightarrow$ normal & shear stresses

Shear stress $\rightarrow 0$ for any fluid at rest, and hence the only surface force on a fluid element is the normal component.

2.3 Pressure on a Stationary Fluid.

Pressure is a measure of force distribution over any surface associated with the force. Pressure may be defined as the force acting along the normal direction on unit area of the surface.

Consider a small wedge fluid element at rest of size (Δx , Δz by ΔS) and depth (b) into the paper by definition there is no shear stress , but we postulate that the pressures p_x, p_z and p_n as shown in Fig. 2.1.

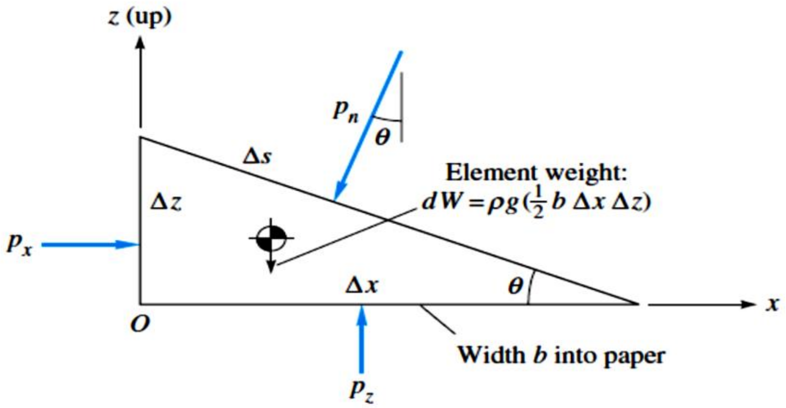


Figure 2.1: Equilibrium of a small wedge of fluid at rest.

Summation of forces must equal zero (no acceleration) in both *x &z* directions

$$\sum Fx = 0 = p_x\, b\ \Delta z - p_n\, b\, \Delta s\, \sin\theta$$

$$\sum Fz = 0 = p_z\, b\, \Delta x - p_n\, b \Delta S\, \cos\theta - \frac{1}{2}\, \gamma\, b \Delta x\, \Delta z \qquad (2.1)$$

But the geometry of the wedge is such that

$$\Delta s\, \sin\theta = \Delta z\ ,\ \Delta s\, \cos\theta = \Delta x \qquad (2..2)$$

Substitution into Eq. (2.1) and rearrangement give

$p_x = p_n \quad , \; p_z = p_n + \frac{1}{2}\gamma\Delta z$ (2.3)

From relation 2.3 illustrate two important principles of hydrostatic

a) There is no pressure change in the horizontal direction.
b) There is a vertical change in pressure proportional to the density, gravity and depth change.

In the limit as the fluid wedge shrinks to a (point) $\Delta z \rightarrow 0$.Then, Eq. 2.3 become

$p_x = p_z = p_n = p$ (2.4)

Since θ is arbitrary

We conclude that the pressure p at a point in a static fluid is independent of orientation. If (*p*) is the hydrostatic pressure using (+ve) sign for tensile, then Eq. 2.4 can be written as

$p = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$ (2.5)

The minus sign accurse because a compression stress is considered to be negative whereas (p) is positive. The pressure is defined as the average of the three normal stresses (σ_{ij}) on the element.

In fluid under static conditions pressure is found to be independent of the orientation of the area. This concept is explained by ***Pascal's law*** which states that the pressure at a point in a fluid at rest is equal in magnitude in all directions.

2.4 Pressure Force on a Fluid Element.

Let the pressure vary arbitrarily $p = p(x,y,z,t)$ consider the pressure acting on the two x-faces as in Fig. 2.2. The net force in the x-direction on the element is given by

$$dF_x = p\,dy\,dz - (p + \frac{\partial p}{\partial x}dx)dy\,dz = -\frac{\partial p}{\partial x}\,dx\,dy\,dz \qquad (2.6)$$

In like manner the net force dF_y involves $-\frac{\partial p}{\partial y}$, and the net force dF_z concerns $-\frac{\partial p}{\partial z}$ the total net –force vector on the element due to pressure is

$$dF_{press} = (-i\frac{\partial p}{\partial x} - j\frac{\partial p}{\partial y} - k\frac{\partial p}{\partial z})\,dx\,dy\,dz \qquad (2.7)$$

Rewrite Eq. 2.7 as the net force per unit element volume and is denoted by (*f*)

$f_{press} = -\nabla p$ (2.8)

This is the pressure gradient causing a net force which must be balanced by gravity or acceleration.

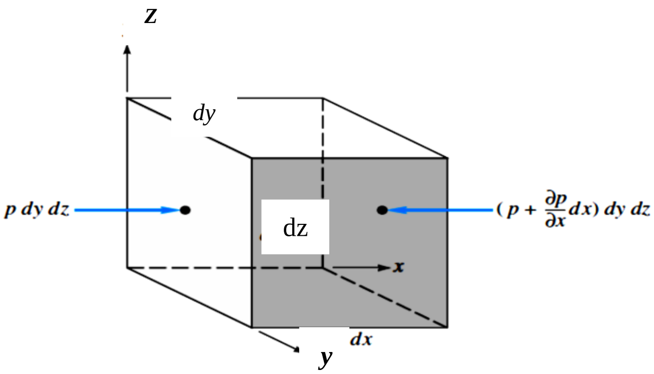


Figure 2.2: Net *x* force on an element due to pressure variation.

The pressure gradient is a surface force which acts on the sides of the element. Also, may be a body force, due to electromagnetic or gravitational potentials acting on the entire mass of the element. Consider only the gravity force or weight of element

Or
$$\left\{ \begin{matrix} dF_{grav} = \rho g\, dx\, dy\, dz \\ f_{grav} = \rho g \end{matrix} \right\} \quad (2.9)$$

For an incompressible fluid with constant viscosity the net viscous force is or (viscous stress)

$$f_{vs} = \mu\left(\frac{\partial^2 \vec{V}}{\partial x^2} + \frac{\partial^2 \vec{V}}{\partial y^2} + \frac{\partial^2 \vec{V}}{\partial z^2}\right) = \mu \nabla^2 \vec{V} \quad (2.10)$$

Where the subscript (vs) stands for viscous force, note that the term ***(g)*** in Eq. 2.9 denotes the acceleration of gravity, a vector acting toward the center of the earth. On earth the average magnitude of ***(g)*** is 32.174 ft/s^2 = 9.807 m/s^2 in our book we use the approximate numerical value of ***g = 32.2 ft/s² = 9.81m/s²***

The total vector resultant of these three forces which are pressure, gravity, and viscous stress must either keep the element in equilibrium or cause it to move with acceleration (**a**). Form Newton's law of motion per unit volume

$$\sum f = \rho \mathbf{a} = f_{press} + f_{grav} + f_{vs} = -\nabla p + \rho g + \mu \nabla^2 V \quad (2.11)$$

Rewrite Eq. 2.11 as follows

$$\nabla p = \rho(g - a) + \mu \nabla^2 V \quad (2.12)$$

Examining Eq. 2.12, we can single out at least four special cases:

1- Flow at rest or at constant velocity: The acceleration and viscous terms vanishes identically, and p depends only upon gravity and density. This is the hydrostatic condition.

2- Rigid – body translation and rotation: The viscous term vanishes identically, and p depends only upon the term ρ(g-a).
3- Irrotational motion $\nabla \times \vec{V} \equiv 0$: The viscous term vanishes identically and exact integral Bernoulli's equation.
4- Arbitrary viscous motion, no general rules apply, but still the integration is quite straight forward.

When the fluid at rest or at constant velocity, ***a*** = 0 and $\nabla^2 V = 0$, Eq. 2.12 for the pressure distribution reduces to

$$\nabla p = \rho g \qquad (2.13)$$

This is a hydrostatic distribution formula and is correct for all fluid at rest. Where ***(g)*** is the magnitude of local gravity, Eq. 2.13 has the pressure components are

$$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma \qquad (2.14)$$

Where the coordinate system ***z*** is up i.e (p) is independent of ***x&y***. Hence $\frac{\partial p}{\partial z}$ can be replaced by the total derivative $\frac{dp}{dz}$ and the hydrostatic condition reduce to

$$\frac{dp}{dz} = -\gamma \qquad (2.15)$$

Equation 2.15 is the fundamental equation for fluids at rest and can be used to determine how pressure change with elevation. This equation indicates that the pressure gradient in the vertical direction is negative; that is, the pressure decrease as we move upward in a fluid at rest.

This leads to the statement,

I. The pressure will be the same at the same level in any connected static fluid and at all points on a given horizontal plane whose density is constant or a function of pressure only.
II. The pressure increases with depth of fluid.
III. The pressure is independent of the shape of the container and the free surface of a liquid will seek a common level in any container, where the free surface is everywhere exposed to the same pressure.

Equation 2.15 is the solution to the hydrostatic problem. The integration requires an assumption about the density and gravity distribution.

2.4.1 Incompressible Fluid.

For liquids the variation in density is usually negligible, even over large vertical distances, so that the assumption of constant specific weight when dealing with liquids is a good one. For this instant, Eq. 2.15 can be directly integrated

$$\int_{p1}^{p2} dp = -\int_{z1}^{z2} \gamma \, dz \quad \text{To yields} \quad p_2 - p_1 = -\gamma(z_2 - z_1)$$

Or $$p_1 - p_2 = \gamma(z_2 - z_1) \quad (2.16)$$

Where $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ are pressures at the vertical elevation $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$ as is illustrated in Fig. 2.3. Eq. 2.16 can be written in compact form

$$p_1 - p_2 = \gamma * h$$

$$or \quad p_1 = p_2 + \gamma * h \quad (2.17)$$

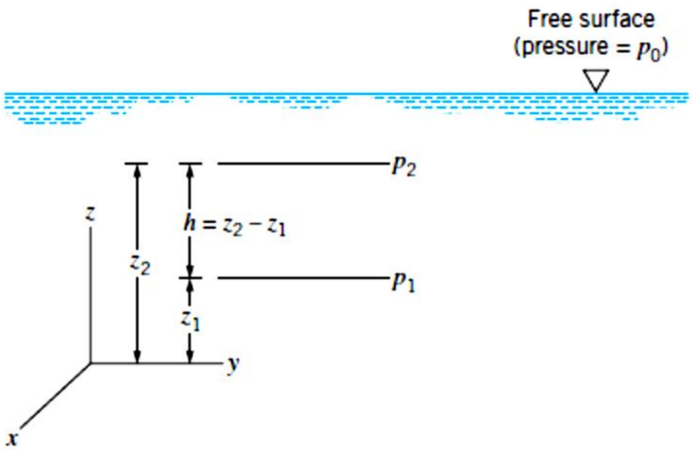


Figure 2.3: Notation for pressure variation in a fluid at rest.

Where ***h*** is the distance, z_2-z_1. This type of pressure distribution is commonly called a *hydrostatic distribution*. Eq. 2.17 shows that in an incompressible fluid at rest the pressure varies linearly with depth. It can also be observed from Eq. 2.17 that the pressure difference between two points can be specified by the distance *h* since

$$h = \frac{p_1 - p_2}{\gamma} \quad (2.18)$$

Where ***h*** is called the pressure head and is interpreted as the height of a column of fluid of specific weight γ to give a pressure difference ($p_1 - p_2$). If $\boldsymbol{p_0}$ is the reference pressure would be the pressure acting on the free surface, then from Eq. 2.17 the pressure at any depth ***h*** below the free surface is given by the following:

$$\boldsymbol{p = p_0 + \gamma h} \quad (2.19)$$

2.4.2 Compressible Fluid.

For compressible fluids such as air, oxygen and other gases where the density can be change significantly with changes in pressure and temperature. Now, Eq. 2.17 can be applying at a point in a gas, it's necessary to consider the possible variation in the specific weights γ before the equation can be integrated. Due to the specific weights of common gases are small when compared with liquids, it follows from Eq. 2.15 that the pressure gradient in the vertical direction is correspondingly small, and even over distances of several hundred meter the pressure will remain essentially constant for a gas. This means we can neglect the effect of elevation changes on the pressure in gases in tanks and pipes. As is described in Chapter 1, the equation of state for an ideal gas is

$$p = \rho RT$$

This relationship can be combined with Eq.2.15 to give

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

By separating variables

$$\int_{p1}^{p2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{T} \qquad (2.20)$$

Where g and R are assumed to be constant over the elevation change from z_1 to z_2. Before completing the integration, one must specify the nature of the variation of temperature with elevation involved. If we assumed that the temperature has a constant value T_o over the range z_1 to z_2 *(isothermal conditions),* it then follows from Eq. 2.20 that

$$p_2 = p_1 \, exp\left[-\frac{g(z_2 - z_1)}{RT_o}\right] \qquad (2.21)$$

Eq. 2.21 provides the desired pressure-elevation relationship for an isothermal layer. For non-isothermal condition a similar procedure can be followed if the temperature-elevation relationship is known. An important application of Eq.2.20 relates to the variation in pressure in the earth's atmosphere. The *standard atmosphere conditions* has been determined that can be used in the design of aircraft, missiles and spacecraft, and in comparing their performance under standard conditions. Several important properties for standard atmospheric conditions at sea level are listed in Table 2.1 and Fig 2.4 shows the temperature decreases with altitude in the region nearest the earth's surface *(troposphere)*, and then becomes essentially constant in the next layer *(stratosphere)*, and subsequently starts to increase in the next layer.

The quantities of the specific weight of fluid γ with dimensions of weight per unit volume are tabulated in Table 2.2.

Table 2.1: Properties of U.S. standard atmosphere at sea level[a][1]

Property	SI Units	BG Units
Temperature, T	288.15 K (15 °C)	518.67 °R (59.00 °F)
Pressure, p	101.33 kPa (abs)	2116.2 lb/ft^2 (abs) [14.696 lb/in.^{2} (abs)]
Density, ρ	1.225 kg/m^3	0.002377 slugs/ft^3
Specific weight, γ	12.014 N/m^3	0.07647 lb/ft^3
Viscosity, μ	1.789×10^{-5} N · s/m^2	3.737×10^{-7} lb · s/ft^2

[a]Acceleration of gravity at sea level = 9.807 m/s^2 = 32.174 ft/s^2.

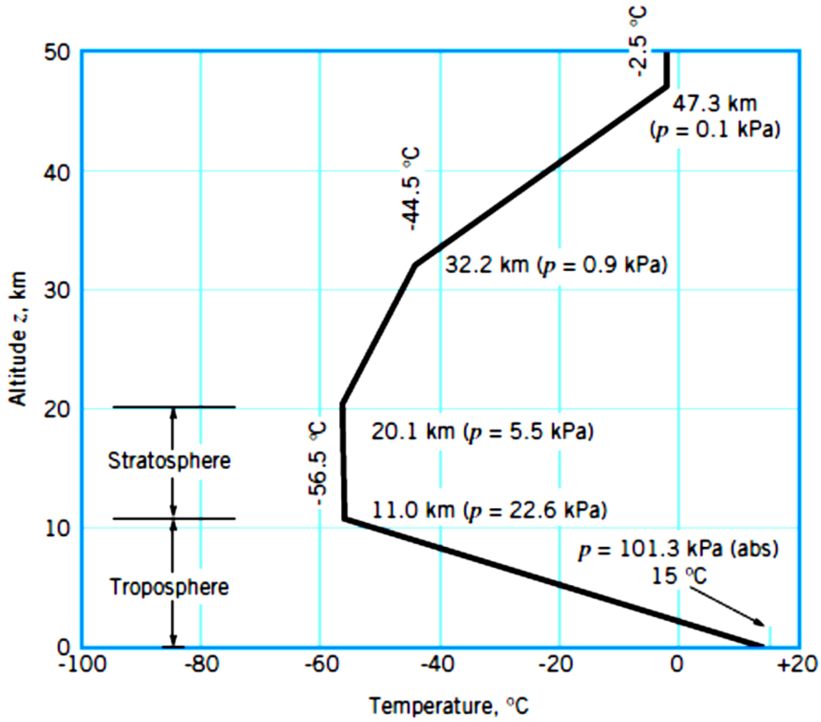


Figure 2.4: Variation of temperature with altitude in the U.S. standard atmosphere [1].

Table 2.2: Specific weight of some common fluids

	Specific weight γ at 68°F = 20°C	
Fluid	**lbf/ft^3**	**N/m^3**
Air (at 1 atm)	0.0752	11.8
Ethyl alcohol	49.2	7,733
SAE 30 oil	55.5	8,720
Water	62.4	9,790
Seawater	64.0	10,050
Glycerin	78.7	12,360
Carbon tetrachloride	99.1	15,570
Mercury	846	133,100

Ex.1

The deepest point in the ocean is (***11034m***) in the pacific. At this depth ***γ=10520*** N/m^3. Estimate the absolute pressure at this depth.

Sol.

$p = p_{atm.} + \gamma * h = 101350 + 10520*11034 = 116179030$ N/m^2

p= 116.18 MPa **Ans.**

Ex.2

A closed tank contains ***1.5 m*** of SAE 30 oil, 1m of water, ***20 cm*** of mercury and an air space on top all at 20°C. If ***p_{bottom}= 60000*** Pa, what is the pressure in the air space. Using the value of γ from Table 2.1

Sol.

Apply the hydrostatic formula down through the three layers of fluid.

$p_{bottom} = p_{air} + \gamma_{oil} * h_{oil} + \gamma_{water} * h_{water} + \gamma_{mercury} * h_{mercury}$

$60000 = p_{air} + (8720\ N/m^3) * (1.5\ m) + (9720\ N/m^3)*(1.0\ m) + (133100\ N/m^3)*(0.2m)$

Solve for the pressure in the air space

p_{air}= 10580 P_a **Ans.**

2.5 Pressure Measurements.

Since pressure is a very important characteristic of a fluid field, it is defined as the force acting along the normal direction on unit area. A more precise mathematical definition of pressure as

$$p = \lim_{A \to a} \left(\frac{\Delta F}{\Delta A}\right) = \frac{dF}{dA} \qquad (2.22)$$

This explicitly means that the pressure is the ratio of the element force to the elemental area (a) normal to it.

The unit of pressure in the SI system is (N/m^2) also called Pascal (Pa). The atmospheric pressure is approximately (10^5 N/m^2) is and designated as "bar". From above definition the pressure at a point within a fluid mass will be designated as either an *absolute* pressure or a *gage* pressure.

Absolute pressure is measured relative to a perfect vacuum (*absolute zero pressure*), where as gage pressure is measured relative to the local *atmospheric pressure.* Thus, a gage pressure of zero corresponds to a pressure that is equal to the local atmospheric pressure. Absolute pressures are always positive, but gage pressure can be either positive or negative depending on whether the pressure is above or below atmospheric pressure. A negative gage pressure is also referred to as a *suction* or *vacuum* pressure. The concept of gage and absolute pressure is illustrated graphically in Fig. 2.5 for two typical pressures located at points 1 and 2. Gage pressure is the difference between the value of the pressure and the local atmospheric pressure ($p_{atm.}$)

$p_{gage} = p - p_{atm.}$

At sea – level, the international standard atmosphere has been chosen as $p_{atm.}$=101.32 (kN/m^2)

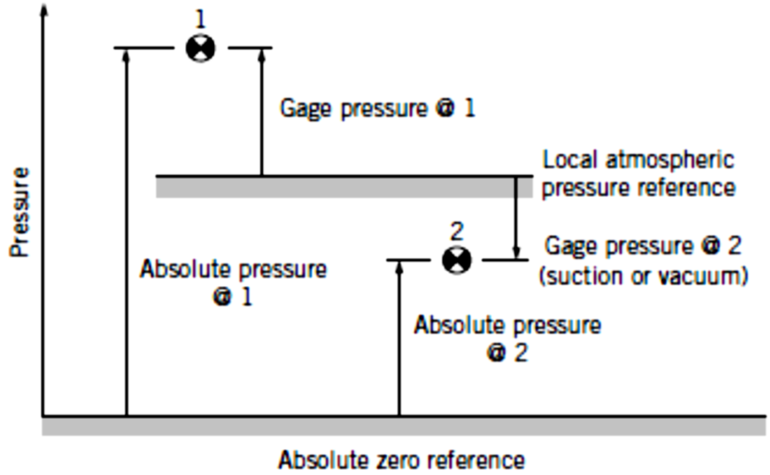


Figure 2.5: Graphical representation of gage and absolute pressure.

The measurement of atmospheric pressure is usually accomplished with a mercury *barometer*, which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in Fig. 2.6. The tube is initially filled with mercury (inverted with its open end up) and then turned upside down (open end down) with the open end in the container of mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor pressure (which develops in the space above the column) balances the force due to the atmospheric pressure. Thus,

$$p_{atm.} = \gamma h + p_{vapor} \qquad (2.23)$$

The vapor pressure p_{vapor} can be neglected in most practical cases in comparison to $p_{atm.}$, since its very small for mercury, p_{vapor}= 0.16*$p_{atm.}$. So that,

$$p_{atm.} = \gamma h$$

$$\therefore\ h = \frac{p_{atm.}}{\rho * g} = \frac{1.0132*10^5 \left(N/m^2\right)}{13560\left(kg/m^3\right)*9.81\left(N/kg\right)} = 0.761\ m\ of\ (Hg)$$

If water was used the value of ***h*** will be equal to ***10.32 m***

Ex.3

What will be the (a) the gauge pressure , (b) the absolute pressure of water at depth 12m below the surface ? ρ_w=1000kg/m^3, p_{atm}=101 kN/m^2.

Sol.

(a) $p_{gage} = \rho g h = 1000 * 9.81 * 12 = 117720 \frac{N}{m^2}, (\boldsymbol{Pa})$

(b) $p_{abs.} = p_{gage} + p_{atm.} = (117720 + 101 * 10^3) = 218720 \frac{N}{m^2} = 218.72 \frac{kN}{m^2}, (kPa)$

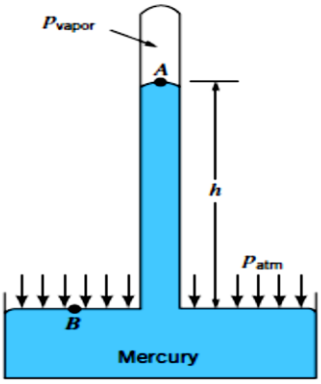


Figure 2.6: Mercury barometer.

2.6 Manometers.

The manometers are the standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called *manometers.* Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

2.6.1 Piezometer Tube.

The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Fig.2.7. Since manometers involve columns of fluids at rest, the fundamental equation describing their use is Eq. 2.19

$$p = p_0 + \gamma h$$

This gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure p_0 and the vertical distance h between p and p_0. Remember that in a fluid at rest pressure will increase as we move downward and will decrease as we move upward. Application of this equation to the piezometer tube of Fig. 2.7 indicates that the pressure p_A can be determined by a measurement of h through the relationship

$$p_A = \gamma_1 h_1$$

The tube is open at the top, the pressure p_0 can be set equal to zero as using a gage pressure, with the height h_1 measured from the meniscus at the upper surface to point (1) then

$$h_1 = \frac{p_A}{\rho g} \tag{2.24}$$

ρ is the working fluid density.

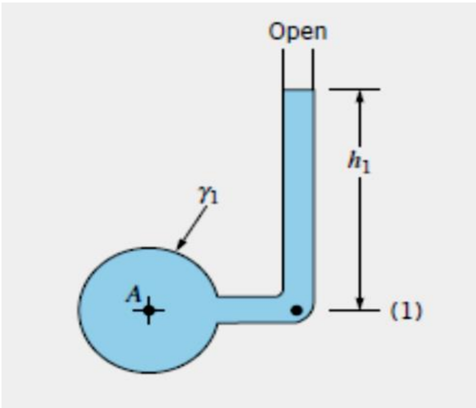


Figure 2.7: Piezometer tube.

2.6.2 U-Tube Manometer.

Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere. Manometer is needed for measuring large gauge pressures. It is basically the modified form of the piezometric tube. A common type manometer is like a transparent "U-tube" as shown in Fig. 2.8.

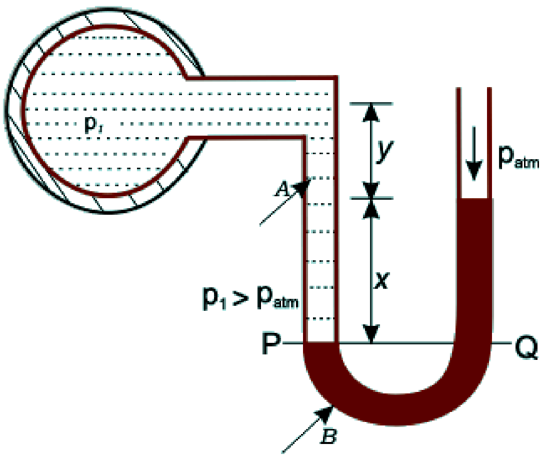


Figure 2.8: A simple manometer to measure gauge pressure [2].

One of the ends is connected to a pipe or a container having a fluid (A) whose pressure is to be measured while the other end is open to atmosphere. The lower part of the U-tube contains a liquid immiscible with the fluid A and is of greater density than that of A. This fluid is called the manometric fluid. The pressures at two points ***P*** and ***Q*** in a horizontal plane as shown in Fig. 2.8 within the continuous expanse of same fluid (the liquid B in this case) must be equal. Then equating the pressures at ***P*** and ***Q*** in terms of the heights of the fluids above those points, with the aid of the fundamental equation of hydrostatics Eq 2.19, we have

$$p_1 + \rho_A g(y + x) = p_{atm} + \rho_B g x$$

Hence $$p_1 - p_{atm} = (\rho_B - \rho_A)gx - \rho_A g y \qquad (2.25)$$

Where p_1 is the absolute pressure of the fluid ***A*** in the pipe or container at its centre line, and p_{atm} is the local atmospheric pressure.

2.6.3 Manometers for Measuring Gauge and Vacuum Pressure.

When the pressure of the fluid in the container is lower than the atmospheric pressure, the liquid levels in the manometer would be adjusted as shown in Fig. 2.9. Hence it becomes,

$$p_1 + \rho_A g y + \rho_B g x = p_{atm}$$

$$p_{atm} - p_1 = (\rho_A y + \rho_B x) * g \qquad (2.26)$$

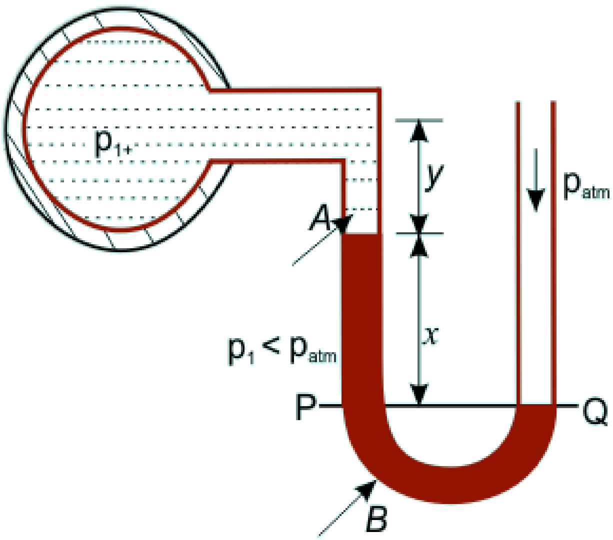


Figure 2.9: A simple manometer to measure vacuum pressure.

2.6.4 Manometers to Measure Pressure Difference.

Another type of manometer is also frequently used to measure the pressure difference, in course of flow, across a restriction in a horizontal pipe as shown in Fig. 2.10. The axis of each connecting tube at A and B should be perpendicular to the direction of flow and also for the edges of the connections to be smooth. Applying the principle of hydrostatics at P and Q we have,

$$p_1 + (y + x)\rho_w g = p_2 + y\rho_w g + \rho_m g x$$

$$p_1 - p_2 = (\rho_m - \rho_w) g x \qquad (2.27)$$

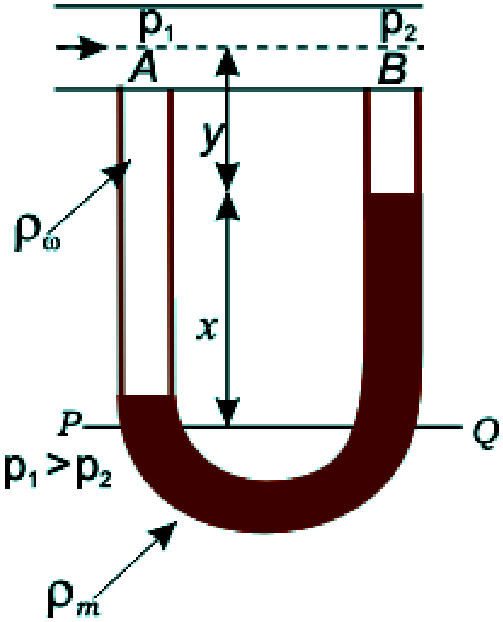


Figure 2.10: Manometer measuring pressure difference [2].

Where, ρ_m is the density of manometric fluid and $\boldsymbol{\rho_w}$ is the density of the working fluid flowing through the pipe.

We can express the difference of pressure in terms of the difference of heads (height of the working fluid at equilibrium).

$$h_1 - h_2 = \frac{p_1 - p_2}{\rho_w g} = \left(\frac{\rho_m}{\rho_w} - 1\right) x \qquad (2.28)$$

Ex.4

A closed tank contains oil and compressed air (***S.G.$_{oil}$ = 0.9***) as is shown in the following figure, a U-tube manometer using mercury (***S.G.$_{Hg}$ = 13.6***) is connected to a tank as shown. For column heights ***h$_1$*=914.5** mm, ***h$_2$*=152.4** mm and ***h$_3$*= 228.6** mm. Determine the pressure reading in ***Pa*** of the gage.

Sol.

The pressure at level (1) is equal to the pressure at level (2), since these two points are at the same elevation in a homogeneous fluid at rest. The pressure at level (1) is

$$p_1 = p_{air} + \gamma_{oil}(h_1 + h_2)$$

The pressure at level (2) is

$$p_2 = \gamma_{Hg}\, h_3$$

Thus, the manometer equation can be expressed as

$$\therefore \quad p_{air} + \gamma_{oil}(h_1 + h_2) - \gamma_{Hg}\, h_3 = 0$$

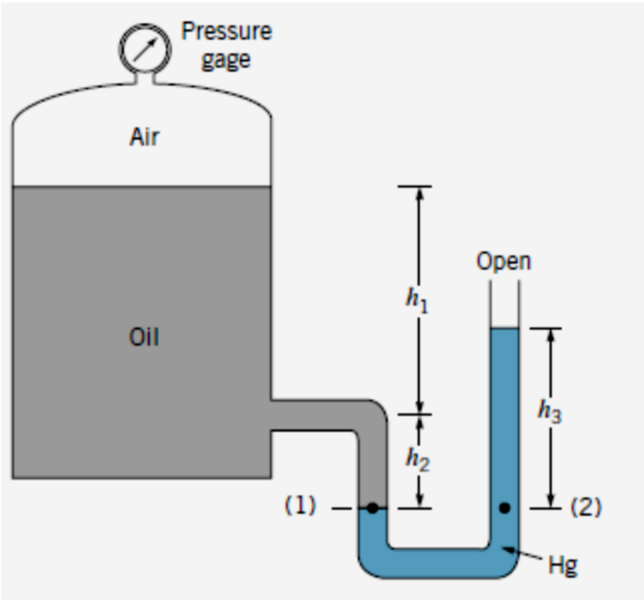
Or

$$p_{air} + S.G_{oil}\,\gamma_{H_2o}(h_1 + h_2) - S.G_{Hg}\gamma_{H_2o}\, h_3 = 0$$

$$p_{air} = -0.9 * 1000 * 9.81 * (0.9145 + 0.1524) + 13.6 * 1000 * 9.81 * 0.2286$$

$$= 21079.23\ N/m^2 (Pa)$$ ***Ans.***

This is the pressure read by the gage, since the specific weight of the air above the oil is much smaller than the specific weight of the oil.



Ex.5

(a)At what depth below the surface of oil, specific gravity is ***0.8*** will produce a pressure of ***120 kN/m²***?.

(b) What depth of water is this equivalent to?.

Sol.

$$S.G. = \frac{\rho_{oil}}{\rho_{water}}, \quad \rho_{oil} = S.G.*\ \rho_{water} = 0.8 * 1000 = 800\ kg/m^3$$

(a) $h = \frac{p}{\rho * g} = \frac{120*10^3}{800*9.81} = 15.29\, m\, of\, oil$ ***Ans.***

(b) $h = \frac{p}{\rho * g} = \frac{120*10^3}{1000*9.81} = 12.23\, m\, of\, water$ ***Ans.***

Ex.6

A manometer connected to a pipe indicates a negative gauge pressure of ***50mm*** of mercury. What is the absolute pressure in the pipe in Newtons per square meter? The atmospheric pressure is 1 bar.

Sol.

$p_{atm.} = 1$ bar $= 1*10^5\ N/m^2$

$p_{abs.} = p_{gage} + p_{atm.}$

$p_{abs.} = \rho * g * h + p_{atm.}$

$p_{abs.} = - 13.6 * 10^3 * 9.81 * 0.05 + 10^5 = 93329$ (Pa)= 93.329 (kPa)

2.6.5 Inverted Tube Manometer.

For the measurement of small pressure differences in liquids, an inverted U-tube manometer is used.

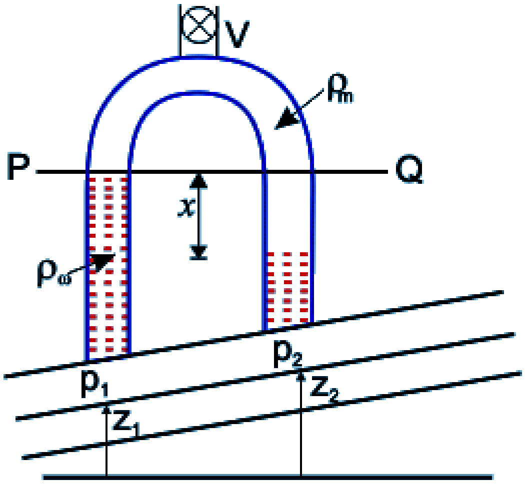


Figure 2.11: An Inverted Tube Manometer

Here $\rho_m < \rho_w$, p at (P & Q) are equal,

$p_1^* - p_2^* = (\rho_w - \rho_m)gx$ Where $p^* = p + \rho g z$

Air is used as the manometric fluid, therefore ρ_m is negligible compared with ρ_w.

$$p_1^* - p_2^* \approx \rho_w g x \qquad (2.29)$$

Air may be pumped through a valve V at the top to reduce the vertical height (***x***) as possible.

2.7 Hydrostatic Forces on Submerged Plane Surface.

Any hydro structure design required a computation of the hydrostatic forces on various solid surfaces contact with fluid. We wish to determine the direction, location and magnitude of the resultant force acting on one side of the surface area due to the liquid in contact. The force acting on ***dA*** (differential area) is $dF = \gamma h\, dA$ which is perpendicular to the surface as shown in Fig. 2.12. The magnitude of the resultant force can be found by summing or integrating this differential force over the entire surface

$$F_R = \int_A \gamma\, h\, dA = \int_A \gamma\, y \sin\theta\, dA$$

Where h =y*sinθ, for constant γ and θ

$$F_R = \gamma\, sin\theta \int_A y\, dA \qquad (2.30)$$

The integral appearing in above equation is the first moment of the area with respect to the x-axis, so we can write

$$\int_A y dA = y_{cg} A$$

$\boldsymbol{y_{cg}}$ is the y- coordinate of the centroid was measured from the x-axis passes through ***cg*** (*the center of gravity*)

$$F_R = \gamma\, A\, y_{cg}\, sin\theta$$

Or more simply as

$$F_R = \gamma\, h_{cg}\, A \qquad (2.31)$$

Note, the magnitude of the force is independent of (θ) and depends only on the specific weight, the total area, and the centroid of the area.

Where ($\boldsymbol{h_{cg}}$) is the vertical distance from the fluid surface to the centroid of the area, but the resultant force is not actually pass through the centroid area. Its line of action passes through the *center of pressure* (***cp***).

The y-coordinate, ($\boldsymbol{y_{cp}}$), of the resultant force can be determined by summation of moments around the x-axis, that is, the moment of the resultant force must equal the moment of the distributed pressure force or

$$F_R\, y_{cp} = \int_A y\, dF = \int_A \gamma\, sin\theta\, y^2 dA$$

And therefore, since $F_R = \gamma\, A\, y_{cg}\, sin\theta$

$$y_{cp} = \frac{\int_A y^2\, dA}{y_{cg}\, A}$$

The integral in the numerator is the second moment of the area (moment of inertia), I_x, thus, we can write

$$y_{cp} = \frac{I_x}{y_{cg} A} = \frac{I_x\, sin\theta}{h_{cg}\, A} \qquad (2.32)$$

From the parallel axis theorem can express I_x, as $I_x = I_{xcg} + Ay_{cg}^2$, where I_{xcg} is the second moment of the area with respect to an axis passing through its centroid and parallel to the x-axis. Thus, from Eq. 2.32

$$y_{cp} = \frac{I_{xcg}}{y_{cg}A} + y_{cg} = \frac{I_{xcg}sin\theta}{h_{cg}A} + y_{cg} \qquad (2.33)$$

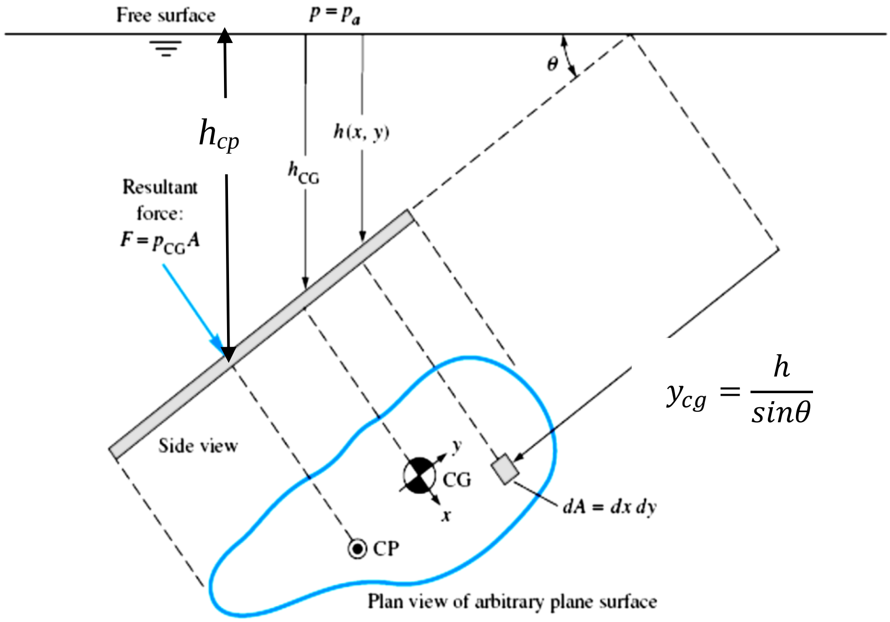


Figure 2.12: Notation for hydro static force on an inclined plane surface of arbitrary shape.

Since $sin\,\theta = \frac{h_{cg}}{y_{cg}}$. From Eq. 2.33 $\frac{I_{xcg}}{y_{cg}A} > 0$, then the resultant force does not pass through the centroid but is always below it.

From Fig. 2.12 $h_{cp} = y_{cp}sin\theta$. Know, from Eq. 2.33

$$h_{cp} = \frac{I_{xcg}sin\theta^2}{h_{cg}A} + h_{cg} \qquad (2.34)$$

If θ=90°, then $sin\theta^2$=1. *i.e.* the surface is vertical. The center of pressure for immersed vertical surface becomes as follows,

$$h_{cp} = \frac{I_{xcg}}{h_{cg}A} + h_{cg} \qquad (2.35)$$

The x-coordinate, $\boldsymbol{x_{cp}}$, for the resultant force can be determined as follows

$$F_R x_{cp} = \int_A x\, dF = \int \gamma \sin\theta\, xy\, dA$$

Since $dF = \gamma \sin\theta\, y\, dA$, $F_R = \gamma A y_{cg} \sin\theta$

Therefore $x_{cp} = \frac{\int_A xy\, dA}{y_{cg} A} = \frac{I_{xy}}{y_{cg} A}$ (2.36)

Where $I_{xy} = I_{xycg} + A\, x_{cg} y_{cg}$ is the product of inertia with respect to the ***x&y*** axes. From parallel-theorem

$$x_{cp} = \frac{I_{xyc}}{y_{cg} A} + x_{cg} \quad (2.37)$$

Summary:-

To find net hydrostatic force on a plane surface:

1- Find area in contact with fluid
2- Locate controid (cg) of that area.
3- Find hydrostatic pressure p_{cg} at controid = $\gamma\, h_{cg}$
4- Find force $F = p_{cg}.A$
5- Location will not be at (c.g.) but at a distance y_{cp} below centroid.

Ex.7

The 4m-diameter circular gate is located in the inclined wall of a large reservoir containing water ***(γ = 9.81 kN/m³)***. The gate is mounted on a shaft along its horizontal diameter. For water depth of (***10 m***) above the shaft determine:

a) The magnitude and location of the resultant force exerted on the gate by the water.
b) The moment that would have to be applied to the shaft to open the gate.

Sol.

The resultant force

$F_R = \gamma h_{cg} A$

$F_R = (9.81 * 10^3)\ \ (10)(4\pi) = 1230 * 10^3\ N = 1.23\ MN$

The location of F_R is at the center of pressure

$x_{cp} = \frac{I_{xyc}}{y_c A} + x_c$

$x_{cp} = 0$ since the area is symmetrical and the (c.p.) must lie along the (A-A) to obtain y_{cp}

$y_{cp} = \frac{I_{xc}}{y_{cg} A} + y_{cg}$

$I_{xc} = \frac{\pi R^4}{4}$, $y_{cg} = \frac{hcg}{sin60}$, $y_{cg} = \frac{10}{sin60}$

$$y_{cp} = \frac{\left(\frac{\pi}{4}\right)(2m)^4}{\left(\frac{10}{sin60}\right)(4\pi)} + \frac{10}{sin60} = 0.0866 + 11.547 = 11.63m$$

The distance below the shaft (along the gate) to the (***cp***)

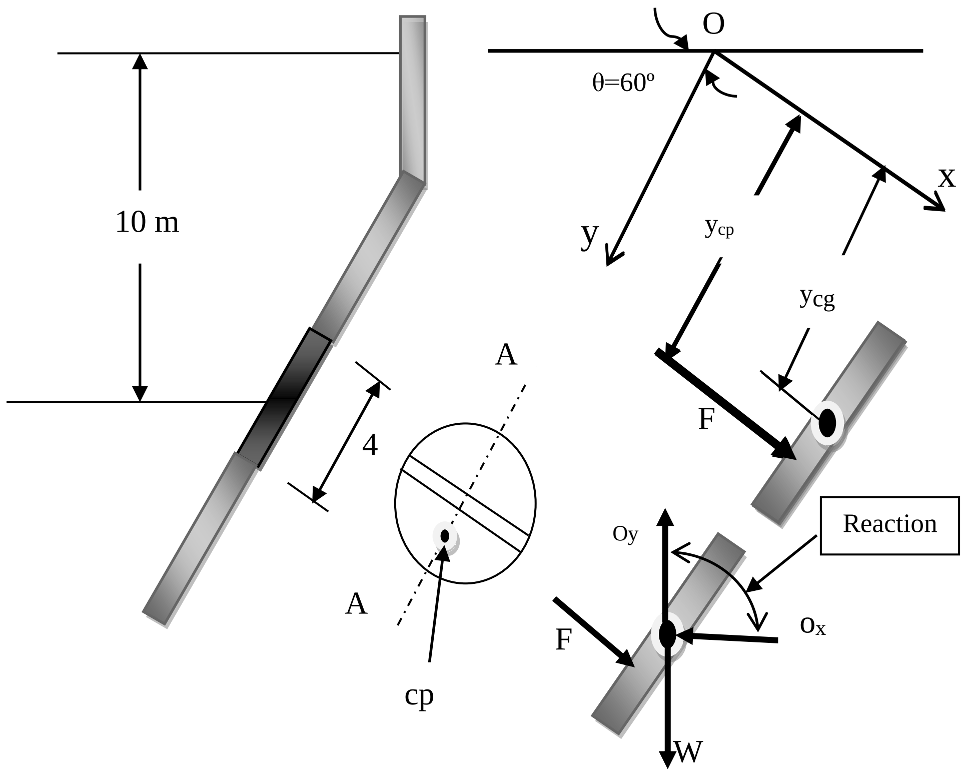
$y_{cp} - y_{cg} = 0.0866\ m \quad Ans(a)$

O_x & O_y are the horizontal and vertical reactions of the shaft on the gate, from the sum moments about the shaft

$\sum M_c = 0$

$M = F_R(y_{cp} - y_{cg})$

$=(1230*10^3)(0.0866) = 1.07 * 10^5$ N.m



2.8 *Hydrostatic Forces on Curved Surface.*

Consider the curved section ***ab*** of the open tank as in Fig. 2.13. We wish to find the resultant force acting on this section with unit length perpendicular to the plane of the paper. The horizontal plane surface ***bc*** and the vertical plane surface ***ac*** are the projection areas of the curved surface ***ab***. $\boldsymbol{F_h}$ **&** $\boldsymbol{F_v}$ are the forces components that the tank exerts on the fluid. γ is the specific weight of the fluid times the enclosed volume acts through (***cg***), then,

i) Vertical forces $\boldsymbol{F_v}$: the vertical force on a curved surface is given by the weight of the liquid enclosed by the surface and the vertical force acts on horizontal free surface of the liquid. The force acts along the center of gravity of the volume.

ii) Horizontal forces $\boldsymbol{F_h}$: the horizontal force equals the force on the projected area of the curved surface and acts at the center of pressure of the projected area.

$F_h=F_2$

$F_v=F_1+W$

The magnitude of the resultant is obtained from the following equation

$$F_R = \sqrt{(F_h)^2 + (F_v)^2} \quad (2.38)$$

The direction of $\boldsymbol{F_R}$ is obtained from the following relation

$$\theta = \tan^{-1}\left(\frac{F_h}{F_v}\right) \quad (2.39)$$

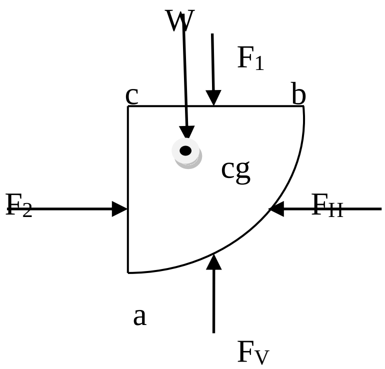


Figure 2.13: Hydrostatic forces on a curved surface

Ex.8

Determine the resultant force exerted by sea water ***S.G.=1.025*** on the curved ***AB*** of an oil tanker as shown in figure. Also determine the direction of action of the force. Consider 1m width perpendicular to paper.

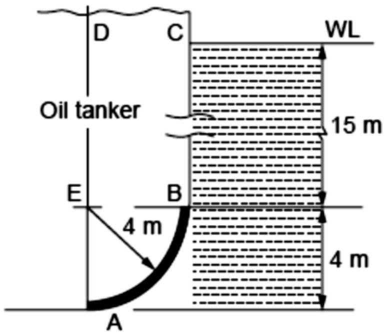
Sol.

$F_h = \gamma * A * h_{cg} = (1025 * 9.81) * (4 * 1)(15 + 4/2) = 683757\, N$

From Eq. 2.35, Line of action of horizontal force

$h_{cp} = \frac{I_{xc}\sin\theta}{h_{cg}A} + h_{cg} = \left[\frac{1\times4^3}{12}\right]\left[\frac{1}{(17\times4\times1)}\right] + 17 = 17.0784m$ from top towards left, The vertical force is due to the volume of sea water displaced

$F_v = [\forall_{BCDE} + \forall_{ABE}]\gamma = [(15 \times 4 \times 1) + (4^2 \times \pi \times 1/4)][1025 \times 9.81] = 729673N$ acts upwards.



To find the location of vertical force which acts at x_{cg} in x-direction;
x_{cg1} of column area *BCDE* is in the vertical plane (*2 m*) from edge.
x_{cg2} of the area *ABE* =(4-4R/3π) = 2.302 m from edge. Taking moments of the area about the edge, the line of action of vertical force is

$X_{cg} = [(x_{cg1}*A_1)+(x_{cg2}*A_2)]/[A_1+A_2] = [2*(15*4) + (2.302*4^2*\pi/4)]/[(15*4)+(4^2\pi/4)]$ = ***2.0523 m*** from the edge.

The resultant force is

$$F_R = \sqrt{(F_h)^2 + (F_v)^2} = \sqrt{(683757)^2 + (729673)^2} = 999973\ N$$

The direction of action to the vertical is,

$$\tan\theta = \frac{F_h}{F_v} = \frac{683757}{729673} = 0.937 \qquad \therefore\ \theta = 43.14°$$

2.9 Buoyancy and Stability of Floating Body.

2.9.1 Buoyancy Force.

The principle of Archimedes is states that, any floating or immersed body in a fluid experiences a vertical bouyant force equal to the weight of the fluid it displaces. The derivation of above princible as follows,

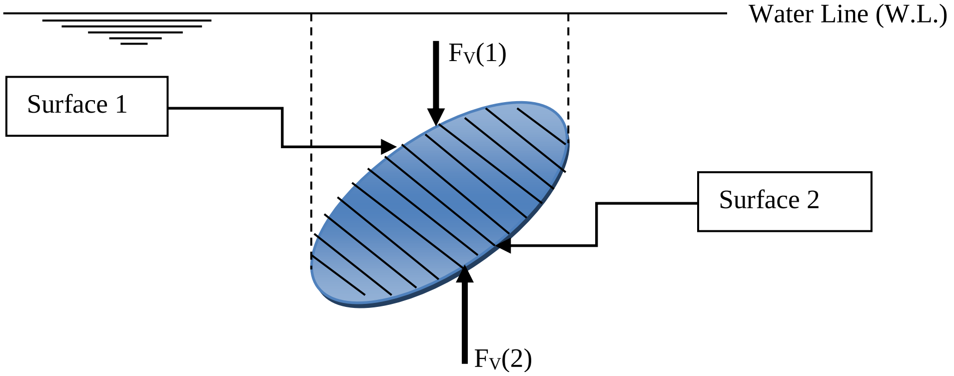


Figure 2.14: Forces on upper and lower curved surface

From Fig. 2.14 the body lies between an upper curved surface (1) and lower surface (2),

- $F_V(1)$= The vertical force of the fluid weight above the surface(1).
- $F_V(2)$= The vertical force of the fluid weight above the surface(2).
- F_B = buoyant force.
- $F_B = F_V(2) - F_V(1)$ = weight of fluid equivalent to body volume.

Now, how to find the vertical force on body?, from Fig 2.15, the sum of vertical forces on elemental vertical slices of immersed body, that can be derived as follows,

$$F_B = \int_{body} (p_2 - p_1) dA_H$$

$$F_B = \gamma \int_{body} (z_2 - z_1) dA_H = \gamma \forall_{body} \quad (2.40)$$

F_B acts at the point is called the center of buoyancy.

Since, ***p_1*** and ***p_2*** are the pressure due to weight of fluid on upper and lower horizontal surface of elemental area

$\forall_{body}$ is the body volume.

p= γz.

z_1 and ***z_2*** are the distances from water line to upper and lower horizontal surface of elemental area.

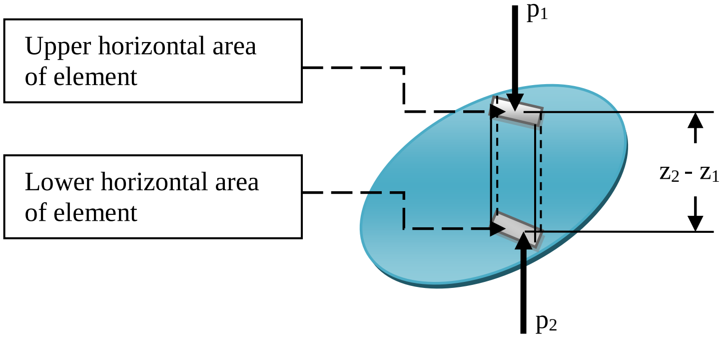


Figure 2.15: Pressures on upper and lower horizontal surface of elemental area

Ex.9

A body is weight ***400N*** in air and its weight ***222N*** in water. Calculate its volume.

Sol.

The summation of forces is

$F_B + T - W = 0$; where T is the tension in cable.

$\therefore F_B = W - T = 400 - 222 = \mathbf{178}\boldsymbol{N}$ weight of displaced fluid.

$F_B = \gamma \times \forall = 9810 \times \forall = 178$ N

$\therefore \forall = \mathbf{0.018}\ \boldsymbol{m^3}$

Ex.10

A spar buoy is a rod weighted to float vertically as in figure. Let the buoy be maple wood (***S.G.=0.6***), its dimension are (***2 in × 2 in × 10 ft***), floating in seawater (***S.G.=1.025***) how many pounds of steel (***S.G.=7.85***) should be added at the bottom so that (***h=18 in***).

Sol.

Let $\forall_{sp.} = wood\ spar\ volume$; $\forall_{Imm.sp.} = Immersed\ spar\ volume$; $W_{st.} = steel\ weight$; $\forall_{st.} = steel\ volume$; $W_{sp} = Wood\ spar\ weight$

$$\forall_{sp.} = \left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(10) = 0.273 ft^3$$

$$W_{st.} = m_{st.} \times g = \rho_{st.} \times \forall_{st.} \times g$$

$$SG_{st.} = \frac{\rho_{st}}{\rho_w} \quad \rightarrow \rho_{st.} = SG_{st.} \times \rho_w \ \ where\ \rho_w = water\ density$$

$$\therefore \forall_{st.} = \frac{W_{st.}}{(SG_{st.})(\gamma_w)} = \frac{W_{st}}{(7.85)(62.4)} \qquad (a)$$

$$\forall_{Imm.Sp.} = \left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(8.5) = 0.236 ft^3$$

From the below figure the buoyant vertical force F_B balances the weights of wood and steel as follows:

$$F_B = W_{sp.} + W_{st.} = (\rho \forall g)_{sp} + W_{st.} = \left(SG \gamma_w \forall_{sp.}\right) + (W_{st.}) \qquad (b)$$

Also, F_B equal to the weight of water displaced by immersed volume

$$F_B = W_{Imm.sp.} + W_{st.} = \left(SG \times \gamma_w \times \forall_{Imm.Sp.}\right) + (SG \times \gamma_w \times \forall_{st.}) \qquad (c)$$

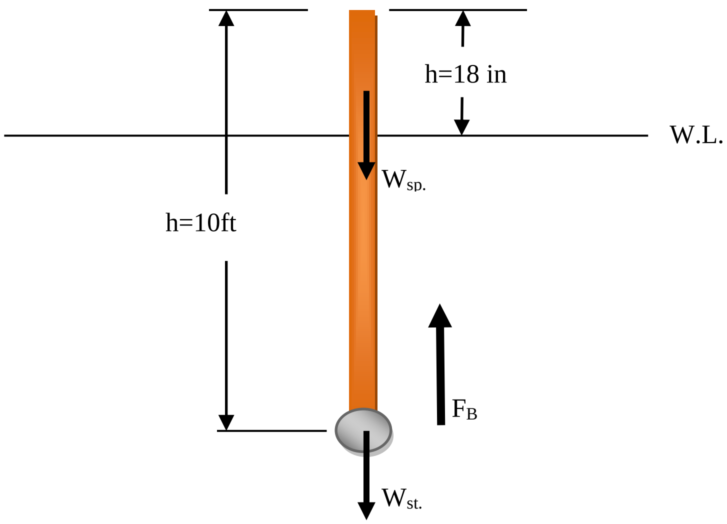
Equating relation ***(b and c)*** and substituting ***Eq. a*** will be given us the following,

$$SG \times \gamma_w \left(\forall_{Imm.Sp.} + \forall_{st.}\right) = \left(SG \gamma_w \forall_{sp.}\right) + (W_{st.})$$

$$(1.025)(62.4)\left[0.236 + \frac{W_{st}}{(7.85)(62.4)}\right] = 0.6 \times 62.4 \times 0.278 + (W_{st.})$$

$15.09 + 0.1306 W_{st.} = 10.4 + W_{st.}$ Solving for $W_{st.}$

$$\therefore W_{st.} = 5.4\ lb_f$$



2.9.2 Stability.

Engineer must design to avoid floating instability; there are three possible situations for a body when immersed in a fluid.

I. If the weight of the body is **greater** than the weight of the liquid of equal volume then the **body will sink into** the liquid (to keep it floating additional upward force is required).

II. If the weight of the body **equals** the weight of equal volume of liquid, then the body will submerge and may **stay at any location** below the surface.

III. If the weight of the body is **less** than the weight of equal volume of liquid, then the body will be partly submerged and **will float in** the liquid.

A ship or a boat should not overturn due to small disturbances but should be stable and return to its original position. Equilibrium of a body exists when there is no resultant force or moment on the body. A body can stay in three states of equilibrium.

i) **Stable equilibrium:** Small disturbances will create a correcting couple and the body will go back to its original position prior to the disturbance.

ii) **Neutral equilibrium:** Small disturbances do not create any additional force and so the body remains in the disturbed position. No further change in position occurs in this case.

iii) **Unstable equilibrium:** A small disturbance creates a couple which acts to increase the disturbance and the body may tilt over completely.

Under equilibrium conditions, two forces of equal magnitude acting along the same line of action, but in the opposite directions exist on a floating/submerged body. These are the gravitational force on the body (weight) acting downward along the centroid of the body and buoyant force acting upward along the centroid of the displaced liquid. Whether floating or submerged, under equilibrium conditions these two forces are equal and opposite and act along the same line.

Fig. 2.16 illustrates the computation for the usual case of a symmetric floating body. The steps are as follows;

1- The basic floating position is calculated from
$F_B = \gamma \forall_{body}$= floating body weight
The body's center of mass at point ***G*** and center of buoyancy ***B*** are computed.

2- After tilted the body at$\boldsymbol{\Delta\theta}$, new position ***B'*** of the center of buoyancy, a vertical line drawn upward from ***B'*** intersects the line of symmetry at point ***M,*** called the metacenter**.** The point about which the body starts oscillating, is called metacenter.

3- If ***M*** is above center of mass where point ***G*** as in figure, the metacentric height $\overline{MG}$ is positive, a restoring moment is present and the original is stable as in Fig. 2.16.b. If $\boldsymbol{M}$ is below $\boldsymbol{G}$, the height $\overline{MG}$ is negative, the body is unstable and the body will overturn as in Fig. 2.16.c. Stable increase with increasing$\overline{MG}$.

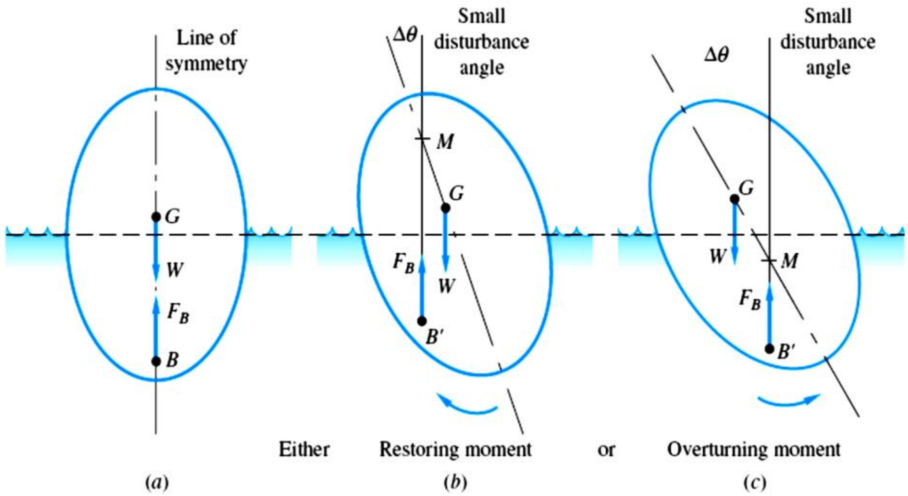
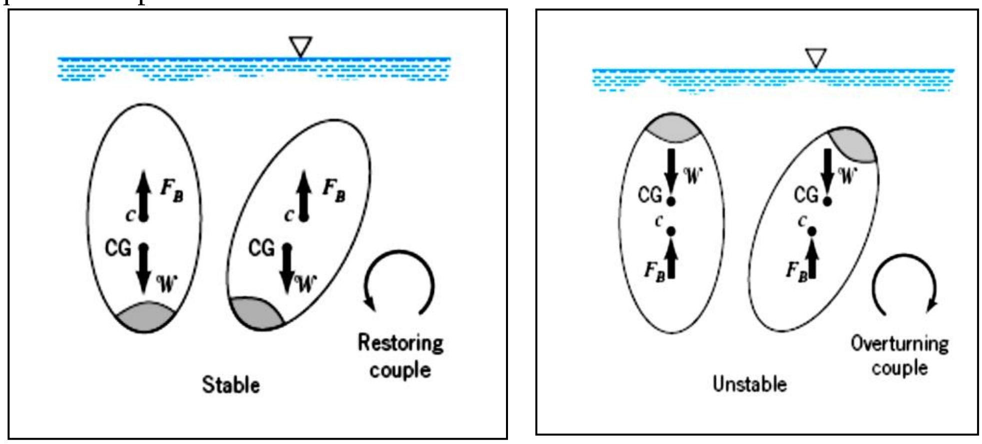


Figure 2.16: The metacenter $\boldsymbol{M}$ of the floating body [1].

Fig. 2.17 below shows the body for completely submerged, which has a center of gravity below the center of buoyancy as in Fig. 2.17.a. For this configuration the body is stable with respect to small rotation. If the center of gravity is above the center of buoyancy as in Fig. 2.17.b, the resulting couple formed by the weight and the buoyant force will cause the body to overturn and to move to a new equilibrium position. Thus, a completely submerged body with its center of gravity above its center of buoyancy is in an unstable equilibrium position.



(a) (b)

Figure 2.17: Stability of a completely immersed body (a) ***CG*** below ***B***, (b) ***CG*** above ***B***

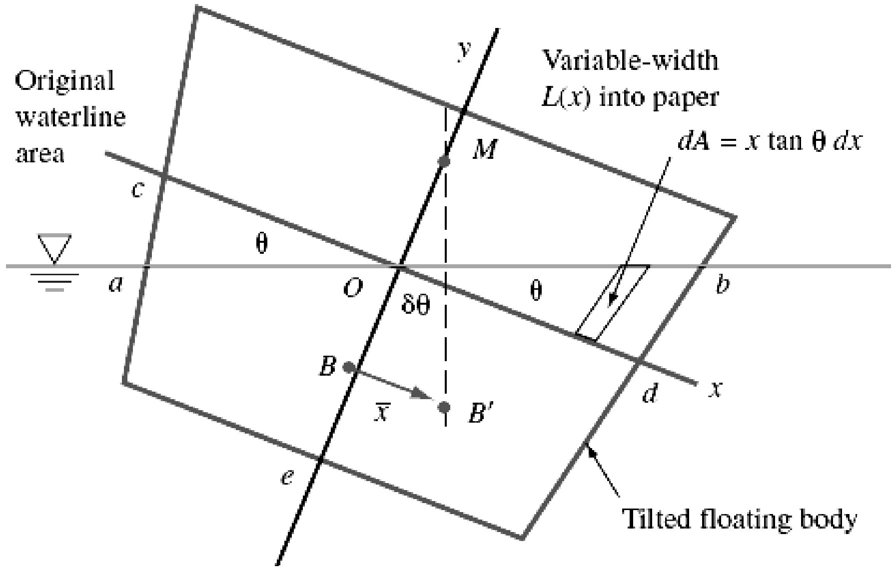


Figure 2.18: A floating body tilted through a small angleθ.

2.9.3 Stability Related to Waterline Area.

From Fig.2.18 the y-axis of the body is assumed to be a line of symmetry. After tilting with small$\boldsymbol{\theta}$, then submerges small wedge ***(obd)*** and uncover and equal wedge ***(coa)*** as in Fig.2.18. New position $\boldsymbol{B'}$ of the center of buoyancy is calculated as the centroid of the submerged position ***(aobde).*** $\bar{x}$ is the movement of the center of buoyancy ***B***, which is related to the waterline area moment of inertia. The moment of inertia of the waterline area calculated as follows:

$$\bar{x}\forall_{aobde} = \int_{codea} x d\forall + \int_{obd} x d\forall - \int_{coa} x d\forall$$

$$\bar{x}\forall_{aobde} = 0(due\ to\ symetry) + \int_{obd} x(LdA) - \int_{coa} x(LdA)$$

$$\bar{x}\forall_{Imm.} = \int_{obd} xL(xtan\theta dx) - \int_{coa} xL(-xtan\theta dx)$$

$$\bar{x}\forall_{Imm.} = tan\theta \int_{W.L} x^2 dA_{W.L} = I_o tan\theta$$

Element of waterline area = Ldx

$$\frac{\bar{x}}{tan\theta} = \overline{MB} = \frac{I_o}{\forall_{Imm.}} = \overline{MG} + \overline{GB} \tag{2.41}$$

Or $\overline{MG} = \frac{I_o}{\forall_{Imm.}} \pm \overline{GB}$; (-) is used if ***G*** above ***B***; (+) is used if ***G*** below ***B***

Where $\boldsymbol{I_o}$ is the area moment of inertia of the waterline footprint of the body about its tilt ***O.*** The computation procedure as follows,

- Firstly determine the distance from ***G*** to ***B***.
- Then make the calculation of $\boldsymbol{I_o}$, and the submerged volume$\forall_{Imm.}$.
- If metacentric height ***MG*** is positive, the body is stable for small disturbances.
- If $\overline{MG}$ negative then the body is unstable.

<u>Ex.11</u>

Consider a wooden cylinder ***S.G.=0.6***, ***1m*** in diameter and ***0.8m*** long. Would this cylinder be stable if placed to float with its axis vertical in oil ***S.G.=0.85.***

<u>Sol</u>.

A vertical force balance gives

$F_B = W_{wood}$

$\gamma_{oil}\forall_{Imm.} = \gamma_{wood}\forall_{wood}$

$0.85 \times 1000 \times 9.81 \times \pi R^2 h = 0.6 \times 1000 \times 9.81 \times \pi R^2 \times 0.8$

$0.85 \times \pi R^2 h = 0.6 \times \pi R^2 \times 0.8 \qquad R = 0.5\ m$

$\therefore h = 0.565\ m$

The point ***B*** is at ***h/2 = 0.282 m*** above the bottom, to predict the metacenter location

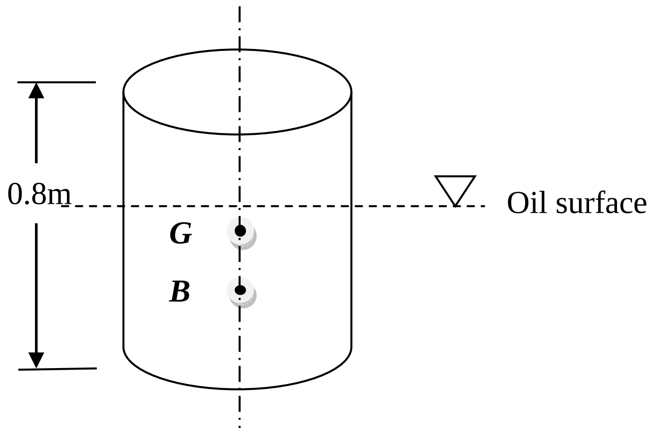
$$MB = {I_o}/{\forall_{Imm.}} = \left[\pi(0.5)^4/4\right]/[\pi \times 0.5^2 \times 0.565] = 0.111m$$

$MB = MG + GB$

Now, GB=0.4-0.282=0.118m from figure.

Hence, MG = 0.111-0.118 = - 0.007m

This float position is thus slightly unstable. The cylinder would turn over.



2.10 Fluid in Rigid-Body Motion.

2.10.1 Acceleration on a Straight Path.

The general equation of motion for fluid that acts as a rigid body (no shear stresses) is determined to be ***Rigid-Body*** motion of fluids.

$\vec{\nabla}p + \rho g\vec{k} = -\rho\vec{a}$ From Eq. 2.12 where the viscous term vanishes identically, and p depends only upon the terms ***ρg &ρa***.

Resolving the vectors into their components,

$$\frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k} + \rho g\vec{k} = -\rho\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$

Accelerating fluids

$$\frac{\partial p}{\partial x} = -\rho a_x;\ \frac{\partial p}{\partial y} = -\rho a_y;\ \frac{\partial p}{\partial z} = -\rho(g + a_z) \qquad (2.42)$$

When the fluid is at rest

$$\frac{\partial p}{\partial x} = 0;\ \frac{\partial p}{\partial y} = 0;\ \frac{\partial p}{\partial z} = -\rho g$$

Considering the container is moving on a straight path with constant acceleration as shown in Fig. 2.19, the ***x&z*** components of acceleration are ***a_x&a_z***, there is no movement in the y-direction and ***a_y***=0. Then the equation of motion for accelerating fluids Eq. 2.42 reduce to

$$\frac{\partial p}{\partial x} = -\rho a_x,\ \frac{\partial p}{\partial y} = 0, and\ \frac{\partial p}{\partial z} = -\rho(g + a_z)$$

Then p=p(x,z), which is

$$dp = \left(\frac{\partial p}{\partial x}\right)dx + \left(\frac{\partial p}{\partial z}\right)dz$$

$$dp = -\rho a_x dx - \rho(g + a_z)dz \qquad (2.43)$$

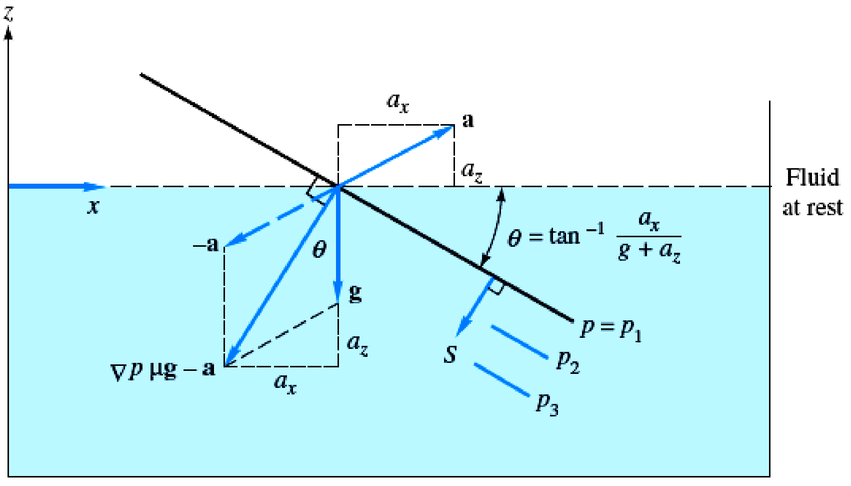


Figure 2.19: Liquid in rigid –body acceleration with constant pressure surface.

By integration Eq. 2.43 along a line of constant pressure between pressure point 1 and 2 when ρ is constant as in Fig. 2.20

$$p_2 - p_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_2) \qquad (2.44)$$

Taking point ***1*** to be origin where x=0 and z=0, since the pressure is atmosphere pressure p_o. The vertical rise or drop of the free surface at point ***2*** relative to point ***1*** can be determine by choosing both ***(1&2)*** on the free surface (so that $p_1=p_2$). Solving Eq. 2.44 for (z_2-z_1).

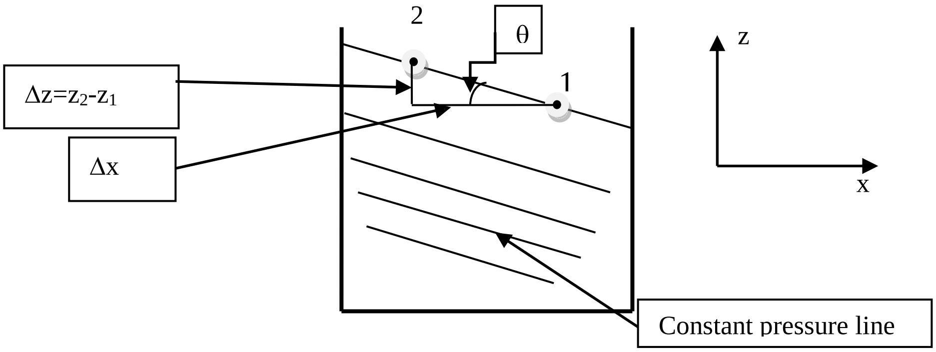


Figure 2.20: Linear acceleration of a liquid with a free surface.

$$\Delta z = z_2 - z_1 = -\frac{a_x}{g+a_z}(x_2 - x_1) \qquad (2.45)$$

Eq. 2.45 is the equation of constant pressure line, called (isobars) obtained from Eq. 2.43. From Fig. 2.20, the line of constant pressure

$$\frac{dz}{dx} = -\frac{a_x}{g+a_z} = constant$$

Slop of isobars is $slop = \frac{dz}{dx} = -\frac{a_x}{g+a_z} = -tan\theta$

$$tan\theta = \frac{a_x}{g+a_z} \qquad (2.46)$$

If $\boldsymbol{a_z=0}$ then $\tan\theta = \frac{a_x}{g}$

Ex.12

An ***80 cm*** high fish tank of cross section (***2m*0.6m***) which is initially filled with water is to be transported on the back of a track. The track accelerates from ***0 to 90 km/hr*** in ***10 s***. If it's desired that no water spills during acceleration, determine the allowable initial water height in the tank. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion?

Sol.

The road is horizontal during acceleration $\therefore a_z = 0$

$$a_x = \frac{\Delta V}{\Delta t} = \left(\frac{90-0}{10}\right)\left(\frac{1}{3.6}\right) = 2.5\ m/s^2$$

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{2.5}{9.81 + 0} = 0.255 \qquad \therefore \theta = 14.3°$$

Then the vertical rise at the back of the tank relative to the mid-plane for two possible orientations as in figure becomes

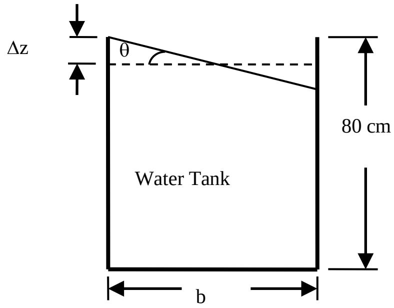
Case-1: The long side is parallel to the direction of motion

$$\Delta z_1 = \left(\frac{b_1}{2}\right)\tan\theta = \left(\frac{2}{2}\right) \times 0.255 = 0.255m = 25.5cm$$

Case-2: The short side is parallel to the direction of motion

$$\Delta z_2 = \left(\frac{b_2}{2}\right)\tan\theta = \left(\frac{0.6}{2}\right) \times 0.255 = 0.076m = 7.6cm$$

The tank should definitely be oriented such that its short side is parallel to the direction of motion, the tank such that its free surface level drop just 7.6 cm, then the initial high becomes = 80-7.6=***72.4cm.***



2.10.2 Rotation in a Cylindrical Container.

This problem is best analyzed in cylindrical coordinates ***(r,θ,z).***the centripetal acceleration of a fluid particle rotating with a constant ω at a distance ***r*** from the axis of rotation is

$a_r = -r\omega^2$ (Directed radially toward the axis of rotation) symmetry about z-axis (axis of rotation) and thus there is no θ dependence as shown in Fig. 2.21.

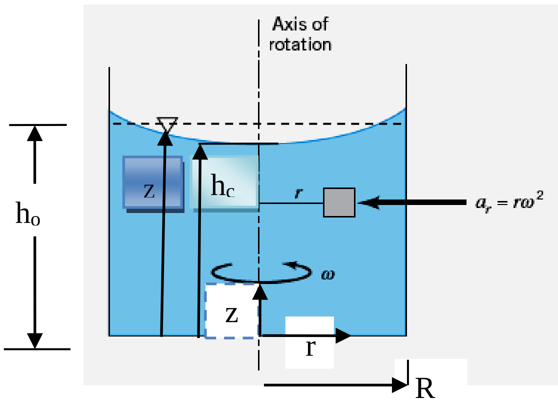


Figure 2.21: Rigid-body rotation of a liquid in a tank.

p=p(r,z); a$_\theta$=0 and ***a$_z$=0*** (no motion in z-direction). Equation of motion for rotating fluids reduce to

$$\frac{\partial p}{\partial r} = -\rho a_r = \rho r\omega^2,\ \frac{\partial p}{\partial \theta} = 0\ \ and\ \ \frac{\partial p}{\partial z} = -\rho g \qquad (2.47)$$

Then the total differential of ***p=p(r,z)***, which is

$dp = \left(\frac{\partial p}{\partial r}\right) dr + \left(\frac{\partial p}{\partial z}\right) dz$, becomes

$$dp = \rho r\omega^2 dr - \rho g dz \qquad (2.48)$$

The equation for surfaces of constant pressure is obtained by setting ***dp=0*** and replace ***z*** by ***z_{isobar}***, which is the value of the surface as function of ***r***, it gives

$$\frac{dz_{isobar}}{dr} = \frac{r\omega^2}{g}$$

By integration the equation for the surface of constant pressure is determined to be

$$z_{isobar} = \frac{\omega^2}{2g} r^2 + C_1 \qquad (2.49)$$

Eq. 2.49 is the equation of parabola. For each isobar surface there is C_1 different. For free surface setting ***r=0*** gives ***$z_{isobar}(0)=C_1=h_c$***. Then Eq. 2.49 for free surface becomes

$$z_s = \frac{\omega^2}{2g} r^2 + h_c \qquad (2.50)$$

Where ***z_s*** is the distance of the free surface. The volume of a cylindrical shell element of radius ***r***, height z_s and thickness ***dr*** is

$$d\forall = 2\pi r z_s dr$$

Then the volume of the parabola formed by the free surface is

$$\forall = \int_{r=0}^{R} 2\pi r z_s dr = 2\pi \int_{r=0}^{R} \left(\frac{\omega^2}{2g} r^2 + h_c\right) r dr$$

$$\forall = \pi R^2 \left(\frac{\omega^2}{4g} R^2 + h_c\right) \qquad (2.51)$$

Original volume in the container is

$$\forall_o = \pi R^2 h_o$$

Where ***h_o*** is the original height, setting these two volumes equal to each other,

$$\pi R^2 h_o = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c\right)$$

$$h_c = h_o - \frac{\omega^2 R^2}{4g}$$

Then Eq. 2.50 for free surface becomes

$$z_s = h_o - \frac{\omega^2}{4g}(R^2 - 2r^2) \qquad (2.52)$$

The maximum height difference is $\Delta z_{s\,max.}$ is at r=R & r=0

$$\Delta z_{s,max} = z_s(R) - z_s(0) = \left(h_o - \frac{\omega^2}{4g}(-R^2)\right) - \left(h_o - \frac{\omega^2}{4g} R^2\right)$$

$$\Delta z_{s,max} = \frac{\omega^2}{4g}R^2 + \frac{\omega^2}{4g}R^2 = \frac{\omega^2}{2g}R^2 \qquad (2.53)$$

For ρ is constant, the pressure difference is determined by integration Eq. 2.48 between two point (1&2) as follows

$$p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

Taking point ***1*** to be the origin ***(r=0, z=0)*** where the pressure is p_o and point ***2*** to be any point in the fluid, the pressure variation is given by

$$p = p_o + \frac{\rho\omega^2}{2}r^2 - \rho g z \qquad (2.54)$$

The pressure is linear in ***z*** and parabolic in ***r***.

Ex.2

A ***20 cm*** diameter, ***60 cm*** high vertical cylinder container shown in figure, is partially filled with ***50 cm*** high liquid whose density is ***850 kg/m³***. Now the cylinder is rotated at a constant speed. Determine the rotational speed at which the liquid will start spilling from the edges of the container.

Sol.

From Eq. 2.52

$$z_s = h_o - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

Taking ***z = 0*** at ***r = 0,*** then the vertical height of the liquid at the edge of container at ***r = R*** becomes

$$z_s(R) = h_o + \frac{\omega^2}{4g}R^2 \; ; \; h_o = 0.5m$$

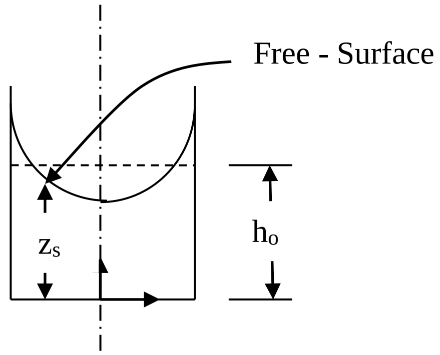
The height of the liquid at edge of the container equals the height of the container, and thus $z_s(R) = 0.6m$ solving for ***ω***

$$\omega = \sqrt{\frac{4g[z_s(R) - h_o]}{R^2}} = \sqrt{\frac{4 \times 9.81 \times (0.6 - 0.5)}{0.1^2}} = 19.8\, rad/s$$

$$\omega = \frac{2\pi n}{60} \quad \to n = \frac{\omega \times 60}{2\pi} = 189\, rpm.$$

The liquid height at the center is

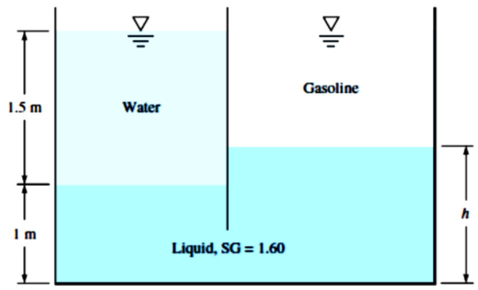
$$z_s(0) = h_o - \frac{\omega^2}{4g}R^2 = 0.4\, m$$



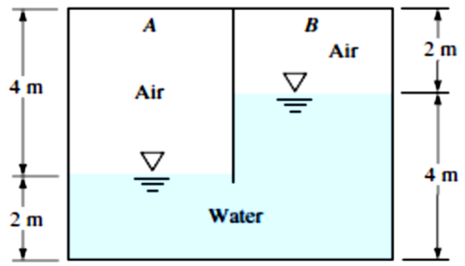
Problems.

P2.1 If a mercury barometer read 700 mm and a bourdon gauge at a point in a flow system reads 500 kN /m^2 what is the absolute pressure at the point. ***[$p_{abs.}$=593.395 kN/m^2]***

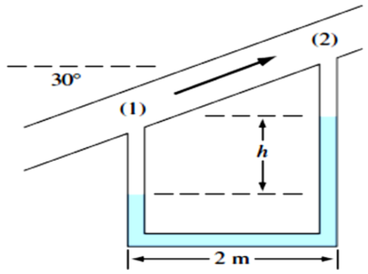
P2.2 As in figure the water & gasoline at ***20C°*** are in the tank open to atmosphere and are at the same elevation what is the height ***(h)*** in third liquid? Take ***γ_{water}=9790 N/m^3 $\gamma_{gasoline}$=6670 N/m^3.*** ***[h=1.52 m]***



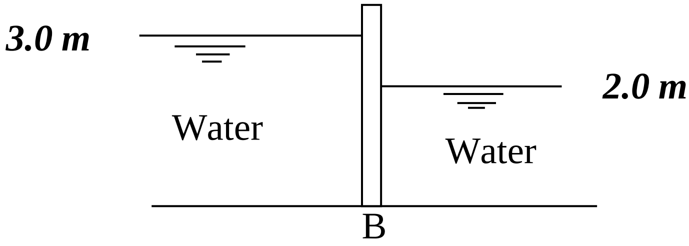
P2.3 The closed tank as in figure is at ***20C°***. If the pressure at (A) is ***95 kPa*** absolute, determine ***(p)*** at B (absolute). What percent error do you make by neglecting the specific weight of the air? ***[p_B=75380 N/m^2 abs. , error%=0.0355]***



P2.4 Water flows upward in a pipe slanted at ***30°***, as in figure the mercury manometer reads ***h=12cm***. What is the pressure difference between points (***1&2***) in the pipe ? ***γ_{water}=9790 N/m^3 $\gamma_{merc.}$=133100 N/m^3***. ***[(p_1-p_2)=26100 pa]***

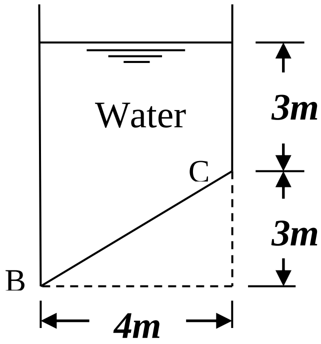


P2.5 A vertical lock gate is ***4m*** wide and separates ***20C°*** water levels ***2m & 3m*** respectively. Find the moment about the bottom required to keep the gate stationary as in below figure. ***[M_B=124 kN.m]***

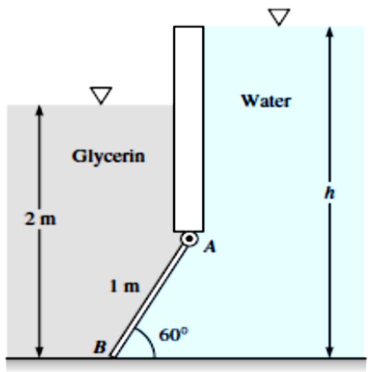


P2.6 The tank as in figure is ***2m*** wide into the paper, neglecting atmospheric pressure, fined the resultant hydrostatic force on panel ***BC***,

a) From a single formula. ***[F=264 kN]***
b) By computing horizontal & vertical force separately, in the spirit of curved surfaces. $\gamma_{water} = 9790\ N/m^3$. ***[$F_R$=441 kN]***



P2.7 Gate ***AB*** as in figure is a homogenous mass of ***180 kg 1.2 m*** wide into the paper, hinged at ***A***, and resting on a smooth bottom at ***B***. All fluids are at 20C°. For what water depth ***h*** will the force at point ***B*** be zero? Take, $\gamma_{water} = 9790\ N/m^3$, $\gamma_{gly.} = 12360\ N/m^3$ ***[h=2.52 m]***

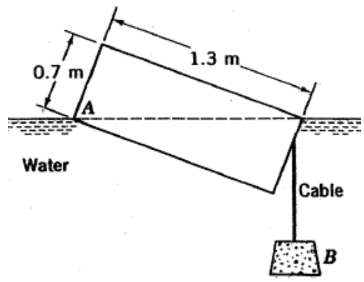


P2.8 Container having ***6.00m*** length, ***1.8m*** height and ***2.1m*** width, filled with water at height ***0.9m***. If the container moves with linear acceleration in length direction of container at ***2.45 m/s²***,

a) Calculate the water force effect on the two ends of container.
b) Explain the difference between these forces equal to unequilibrium force required to accelerate the liquid mass.
c) If the container filled completely by water to ***1.8m*** height and accelerated to ***1.52 m/s²***. Calculate the volume of water is escaping from container in litter.

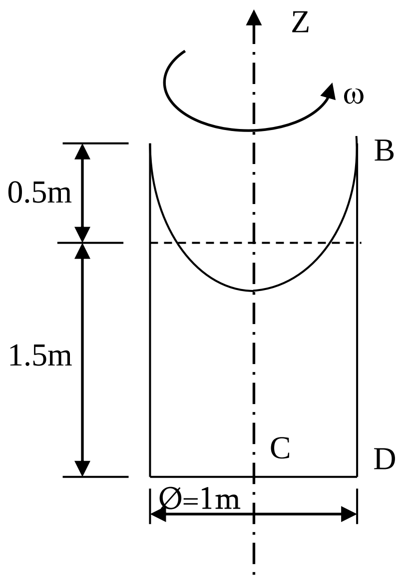
[a-F_{front}=231N, F_{rear}=28043 N, b-$F_{req.}$=27812 N, c- $\forall_{out} = 5860$ l]

P2.9 The homogeneous wooden block **A** as in the figure with dimensions ***(0.7m*0.7m*1.3m)*** and its weight **2.4 kN**. The concrete block **B** (specific weight = 23.6 kN/m^3) is suspended from **A** by cable causing **A** to float in the position indicated. Determine the volume of **B**.
[$\forall = \mathbf{0.0522\ m^3}$]



P2.10 Cylindrical reservoir open from top its height ***2m*** and diameter ***1m*** contain ***1.5m*** of water, if the cylindrical shape rotate about its geometrical center

a) What is the constant angular velocity can be reaches without escaping the waters from reservoir? ***[ω=8.86 rad/s]***
b) What is the pressure at the base of container at ***C & D*** as in figure if ***ω=6.0rad/s***? ***[p_C=12500 N/m², p_D=17000 N/m²]***



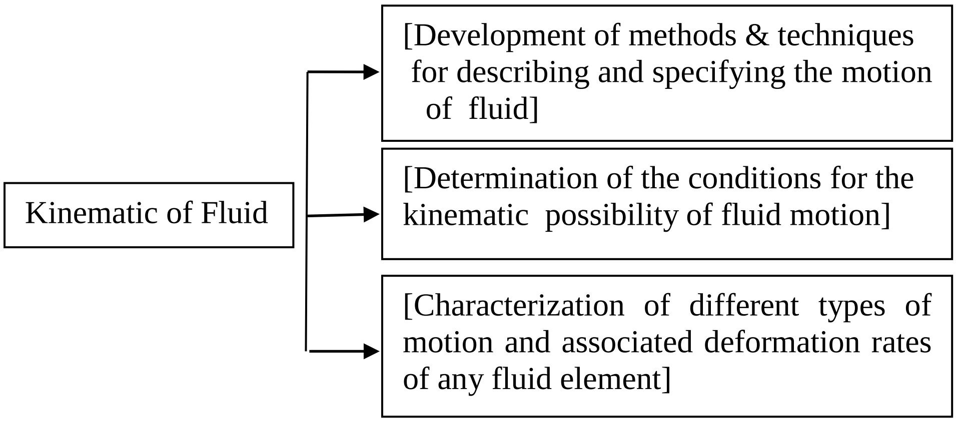
CHAPTER 3

Fluid Flow-Basic Concept

3.1 Definitions.

A. *Kinematics of Fluid.* Is the geometry of motion, which is describes the fluid motion and its consequences without consideration of the nature of forces causing the motion.

The subject has three main aspects



B. Ideal Fluid. Is a frictionless and incompressible fluid, the flow processes are reversible and nonviscous.

C. Laminar Flow. When the fluid particles move along smooth paths in laminas, or layers with one layer gliding smoothly over an adjacent layer this flow is called laminar.

D. Turbulent Flow. The fluid particles are moving in very irregular paths causing an exchange of momentum from one portion of the fluid to another.

E. Scalar & vector fields.

Scalar: - scalar is a quantity which can be expressed by a single number representing its magnitude, as mass, density, and temperature.

Scalar field: **-** If at every point in a region, a scalar function has a defined value, the region is called a scalar field, as temperature distribution in a rod.
Vector:- Vector is a quantity which is specified by magnitude and direction, as force, velocity and displacement.
Vector field: - If at every point in a region, a vector function has a defined value, the region is called a vector field, as, velocity field of a flowing fluid.

F. *Flow field:* - The region in which the flow parameters, as velocity, pressure etc, are defined at each and every point at any instant of time is called a flow field.

3.2 Description of Fluid Motion.

a. Lagrangian Method (L.M.).

The fluid motion is described by tracing the kinematic behavior of each particle constituting the flow. This method depends on the identities of the particles are made by specifying their initial position (spatial location) at a given time. The position of a particle at any other instant of time then becomes a function of its identity and time.
Analyticaly can be expressed as

$$\vec{S} = S(\vec{S}_0, t)$$

$\vec{S}$ is the position vector of a particle with respect to a fixed point of reference at a time (t).
$\overrightarrow{S_0}$ Its initial position at a given time $t=t_0$
The above equation can be written into scalar components with respect to a rectangular cartesian frame of coordinates as:

$$X=X(x_0,y_0,z_0,t) \quad (3.1.a)$$

$$Y=Y(x_0,y_0,z_0,t) \quad (3.1.b)$$

$$Z=Z(x_0,y_0,z_0,t) \quad (3.1.c)$$

Where, x_0, y_0, z_0 are the initial coordinates and x, y, z are the coordinates at time t of the particles
Hence, $\vec{S}$ can be expressed as
$\vec{S}= \vec{\imath}x + \vec{\jmath}y + \vec{k}z$ where $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ are the unit vectors along x ,y and z axes respectively .
The velocity $\vec{V}$ and acceleration $\vec{a}$ of the fluid particle can be obtained from the material derivatives of the position of the particle with respect to time. Therefore,

$$\vec{V} = \left[\frac{ds}{dt}\right]_{s0} \quad (3.2)$$

In terms of scalar components

$$u = \left(\frac{dx}{dt}\right)_{x0,y0,z0} \quad (3.2.a)$$

$$v = \left(\frac{dy}{dt}\right)_{x0,y0,z0} \quad (3.2.b)$$

$$w = \left(\frac{dz}{dt}\right)_{x0,y0,z0} \quad (3.2.c)$$

Where u , v , w are the components of velocity in x , y, z direction respectively.

Similarly, for the acceleration

$$\vec{a} = \left[\frac{d^2\vec{S}}{dt^2}\right] \quad (3.3)$$

Hence ,

$$a_x = \left[\frac{d^2x}{dt^2}\right]_{x0,y0,z0} \quad (3.3.a)$$

$$a_y = \left[\frac{d^2y}{dt^2}\right]_{x0,y0,z0} \quad (3.3.b)$$

$$a_z = \left[\frac{d^2z}{dt^2}\right]_{x0,y0,z0} \quad (3.3.c)$$

Where a_x , a_y, a_z are the acceleration in x , y and z direction respectively.

Advantage of L.M

1- The motion & trajectory of each fluid particle is known.

2- The particles are identified at the start and traced throughout their motion.

Disadvantage: the solution of the equations presents appreciable mathematical difficulties.

b. Eulerian Method.

It avoids the determination of the movement of each individual fluid particle in all details. It seeks the velocity $\vec{V}$ and its variation with time t at each and every location $\vec{s}$ in the flow field. Mathematical representation of the flow field in Eulerian method

$$\vec{V} = V(\vec{S}, t) \quad (3.4)$$

where

$$\vec{V} = \vec{\imath}u + \vec{\jmath}v + \vec{k}w \quad and \quad \vec{S} = \vec{\imath}x + \vec{\jmath}y + \vec{k}z$$

Therefor

u=u(x ,y ,z ,t)

v=v(x ,y ,z ,t)

w=w(x ,y ,z ,t)

3.3 Variation of Flow Parameters in Time & Space.

Hydrodynamic parameters like pressure and density along with flow velocity may vary from one point to another and also from one instant to another at a fixed point. According to type of variation

A- ***Steady flow***. A steady flow is defined as a flow in which the various hydrodynamic parameters and fluid properties at any point do not change with time.

$$\partial\rho/_{\partial t} = 0, \quad \partial p/_{\partial t} = 0, \quad \partial T/_{\partial t} = 0$$

In Eulerian approach a steady flow is described as

$\vec{V} = V(\vec{S})$And $\vec{a} = a(\vec{S})$

The hydrodynamic parameters may vary with location, but the spatial distributions of these parameters remain invariant with time. In Lagrangian approach, the velocities of all points passing through any fixed point at different times will be same. Therefore, the Eulerian and Lagrangian approach of describing fluid motion become identical under this situation.

B- ***Unsteady flow.*** Is defined as a flow in which the hydrodynamic parameters and fluid properties changes with time.

$$\partial\rho/_{\partial t} \neq 0, \quad \partial p/_{\partial t} \neq 0, \quad \partial T/_{\partial t} \neq 0$$

C- ***Uniform flow.*** the flow is defined as uniform flow when in the flow field the velocity and other hydrodynamic parameters do not change from point to point at any instant of time can be expressed as : v=v(t)

Any hydrodynamic parameter will have one value in the entire field

If changes with time → unsteady uniform flow

OR

Does not change with time → steady uniform flow

D. ***Non–uniform flow.*** When the velocity other hydrodynamic parameters changes from one point to another the flow is defined as non- uniform. Non- uniform may be found either in the direction of flow or in direction perpendicular it.

3.4 Material Derivative and Acceleration.

The velocity components u,v,w of the particle along x, y and z direction in space , can be written in Eulerian form as

u=u (x , y, z)

v=v(x , y, z)

z=z(x , y, z)

After an infinitesimal time interval t, let the particle move to a new position given by the coordinates (x+Δx,y+Δy,z+Δz)

Its velocity components at this new position be (u+Δu ,v+Δv, w+Δw).

Expression of velocity components in the Taylor's series form

$$u + \Delta u = u(x, y, z, t) + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z + \frac{\partial u}{\partial t}\Delta t +$$

$$higher\ order\ terms\ \ in\ \Delta x, \Delta y, \Delta z \& \Delta t.$$

$$v + \Delta v = v(x,y,z,t) + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y + \frac{dv}{dz}\Delta z + \frac{\partial v}{\partial t}\Delta t + \text{higher order termsin } \Delta x, \Delta y, \Delta z \& \Delta t.$$

$$w + \Delta w = w(x,y,z,t) + \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z + \frac{\partial w}{\partial t}\Delta t + \text{higher order terms in } \Delta x, \Delta y, \Delta z \& \Delta t.$$

The increment in space coordinates can be written as:-

$\Delta x = u\Delta t,\ \Delta y = v\Delta t, and\ \Delta z = w\Delta t$

Substituting the value of Δx, Δy, Δz in above eqn. , we have

$$\frac{\Delta u}{\Delta t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\ , etc$$

In the limit Δt→ 0 , the equation becomes

$$\left.\begin{aligned}\frac{Du}{Dt} &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\\ \frac{Dv}{Dt} &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\\ \frac{Dw}{Dt} &= \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\end{aligned}\right\} \qquad (3.5)$$

The above Eq's. tell that the operator for total differential with respect to time, D/Dt in a convective field is related to the partial differential as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \qquad (3.6)$$

Material or substantial derivative   Temporal or local derivative   convective derivative

From Eq. 3.5

- The terms in the left hand sides in (x,y,z) are the component of substantial or material acceleration.
- The first terms in the *R.H.S* are the local or temporal accelerations
- While the other terms are convective accelerations.

Thus we can write,

$$\begin{aligned}a_x &= \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\\ a_y &= \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\\ a_z &= \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\end{aligned} \qquad (3.7)$$

Material or substantial acceleration = temporal acceleration + convective acceleration, The total acceleration vector is

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \qquad (3.8)$$

Ex.1

Given the velocity field

$\vec{V} = (4 + xy + 2t)\vec{i} + 6x^3\vec{j} + (3xt^2 + z)\vec{k}$

Find the acceleration of fluid particles

a- As function of x ,y , z and t
b- At (1,1,1) and time t=1sec.

Sol.

a- From the given velocity field

$u = 4 + xy + 2t, \quad v = 6x^3, \quad w = 3xt^2 + z$

From Eq. 3.7

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial t} = 2 \, ; \frac{\partial u}{\partial x} = y \, ; \frac{\partial u}{\partial y} = x \, ; \frac{\partial u}{\partial z} = o$$

$$a_x = 2 + (4 + xy + 2t)(y) + (6x^3)(x) + (3xt^2 + z)(0)$$

$$a_x = 2 + 4y + xy^2 + 2ty + 6x^4 \qquad (a)$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$\frac{\partial v}{\partial t} = 0 \, ; \frac{\partial v}{\partial x} = 18x^2 ; \frac{\partial v}{\partial y} = 0 ; \frac{\partial v}{\partial z} = 0$$

$$a_y = 0 + (4 + xy + 2t)(18x^2) + (3xt^2 + z)(0)$$

$$a_y = 72x^2 + 18yx^3 + 36tx^2 \qquad (b)$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial t} = 6xt; \quad \frac{\partial w}{\partial x} = 3t^2 \, ; \quad \frac{\partial w}{wy} = 0; \quad \frac{\partial w}{\partial z} = 1$$

$$a_z = 6xt + (4 + xy + 2t)(3t^2) + (6x^3)(0) + (3xt^2 + z)(1)$$

$$a_z = 6xt + 12t^2 + 3xyt^2 + 6t^3 + 3xt^2 + z \qquad (c)$$

Combining Eq's (a , b &c) the total acceleration as in Eq. 3.8

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{a} = (2 + 4y + xy^2 + 2ty + 6x^4)\vec{i} + (72x^2 + 18x^3y + 36tx^2)\vec{j} + (16xt + 12t^2 + 3xyt^2 + 6t^3 + 3xt^2 + z)\vec{k}$$

b-

At 1,1,1 and t=1 acceleration vector is

$$\vec{a} = (2 + 4 + 1 + 2 + 6)\vec{i} + (72 + 18 + 36)\vec{j} + (6 + 12 + 3 + 6 + 3 + 1)\vec{k}$$

$$\vec{a} = 15\vec{i} + 126\vec{j} + 31\vec{k}$$

Ex.2

In a fluid flow, the velocity field is given by

$\vec{V} = (3x + 2y)\vec{i} + (2z + 3x^2)\vec{j} + (2t - 3z)\vec{k}$

Determine

a) The velocity components u , v ,w at any point in the flow field

b) The speed at point (1 , 1 , 1)
c) The speed at time t= 2s at point (0,0,2)
d) Classify the velocity field as steady or unsteady, uniform or non uniform and one two or three dimensional.

Sol.

From the given velocity field

a) Velocity components are :

$u = 3x + 2y; \quad v = (2z + 3x^2)\ ;\ w = (2t - 3z)$

b) Speed at point (1,1,1); $\vec{V}_{(1,1,1)}$

Substituting x=1, y=1, z=1 in the expression for u, v & w.

$u = (3 + 2) = 5\ , v = (2 + 3) = 5\ \ , w = (2t - 3)$

$V^2 = u^2 + v^2 + w^2 = 5^2 + 5^2 + (2t - 3)^2 = 25 + 25 + 4t^2 - 12t + 9$

$V^2 = 4t^2 - 12t + 59 \quad \rightarrow \quad V = \sqrt{4t^2 - 12t + 59}$

c) Speed at t= 2s at point (0,0,2):

Substituting t=2, x=0, y=0, z=2 in the expression of u, v & w we get,

$u = 0\ , \quad v = (2 * 2) = 4, \ \ w = (2 * 2 - 3 * 2) = -2$

$V^2 = u^2 + v^2 + w^2 = 0 + 4^2 * (-2)^2 = 20$

Or $V_{(0,0,2)} = \sqrt{20} = 4.472\ \ units$

Velocity field types

i) Since $\vec{V}$ at given (x,y,z) depends on t, it's unsteady flow
ii) Since at given t velocity changes in x direction it's non-uniform.
iii) Since $\vec{V}$depends on x, y, z; its three dimension flow.

Ex.3

Velocity for a two dimensional flow field is given by

$\vec{V} = (3 + 2xy + 4t^2)\vec{\imath} + (xy^2 + 3t)\vec{\jmath}$

Find the velocity and acceleration at a point (1, 2) after 2s.

Sol.

Velocity $\vec{V}_{(1,2)}$

Substituting x=1, y=2 and t=2 in the expression of velocity field, we get

$\vec{V} = (3 + 2 * 1 * 2 + 4 * 2^2)\vec{\imath} + (1 * 2^2 + 3 * 2)\vec{\jmath} = 23\vec{\imath} + 10\vec{\jmath}$

$\therefore \vec{V}_{(1,2)} = \sqrt{23^2 + 10^2} = 25.08\, units.$

Acceleration at point (1,2), $a_{(1,2)}$

We know that $\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + \left(u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y}\right)$

$\frac{\partial\vec{V}}{\partial x} = 2y\vec{\imath} + y^2\vec{\jmath}$

$\frac{\partial\vec{V}}{\partial y} = 2x\vec{\imath} + 2xy\vec{\jmath}$

$\frac{\partial\vec{V}}{\partial t} = 8t\vec{\imath} + 3\vec{\jmath}$

$\vec{a} = (3 + 2xy + 4t^2)(2y\vec{\imath} + y^2\vec{\jmath}) + (xy^2 + 3t)(2x\vec{\imath} + 2xy\vec{\jmath}) + (8t\vec{\imath} + 3\vec{\jmath})$

$\vec{a} = (3 + 2 * 1 * 2 + 4 * 2^2)(2 * 2\vec{\imath} + 2^2\vec{\jmath})$
$\quad + (1 * 2^2 + 3 * 2)(2 * 1\vec{\imath} + 2 * 1 * 2\vec{\jmath}) + (8 * 2\vec{\imath} + 3\vec{\jmath})$

$\vec{a} = 23(4\vec{\imath} + 4\vec{\jmath}) + 10(2\vec{\imath} + 4\vec{\jmath}) + (16\vec{\imath} + 3\vec{\jmath})$

$\vec{a} = 92\vec{\imath} + 92\vec{\jmath} + 20\vec{\imath} + 40\vec{\jmath} + 16\vec{\imath} + 3\vec{\jmath} = 128\vec{\imath} + 135\vec{\jmath}$

$a_{1,2} = \sqrt{128^2 + 135^2} = 186.03\, units$

Ex.4

Find the velocity and acceleration at a point (1, 2, 3) after 1s for a three-dimensional flow given by

$u = yz + t, \qquad v = xz - t, \qquad w = xy$

Sol.

Given; three-dimensional flow field velocity at a point 1,2,3 V(1,2,3) after 1 sec. is

$\mathrm{u} = \mathrm{yz} + \mathrm{t} = 2 * 3 + 1 = 7\frac{\mathrm{m}}{\mathrm{s}}$

$\mathrm{v} = \mathrm{xz} - \mathrm{t} = 1 * 3 - 1 = 2\frac{\mathrm{m}}{\mathrm{s}}$

$w = xy = 1 * 2 = 2\frac{m}{s}$

$\vec{V}_{(1,2,3)} = 7\vec{\imath} + 2\vec{\jmath} + 2\vec{k}$

$V = \sqrt{7^2 + 2^2 + 2^2} = 7.55\frac{m}{s}$

Acceleration $\vec{a}_{(1,2,3)}$

Now $V = (yz + t)\vec{\imath} + (xz - t)\vec{\jmath} + xy\vec{k}$

Acceleration

$\vec{a} = \frac{d\vec{V}}{dt} = \left(u * \frac{\partial\vec{V}}{\partial x} + v * \frac{\partial\vec{V}}{\partial y} + w * \frac{\partial\vec{V}}{\partial z}\right) + \frac{\partial\vec{V}}{\partial t}$

$\vec{a} = (yz + t) * \left(z\vec{\jmath} + y\vec{k}\right) + (xz - t)\left(z\vec{\imath} + x\vec{k}\right) + xy(y\vec{\imath} + x\vec{\jmath}) + (1\vec{\imath} - 1\vec{\jmath})$

$\vec{a}_{(1,2,3)} = 7\left(3\vec{\jmath} + 2\vec{k}\right) + 2\left(3\vec{\imath} + 1\vec{k}\right) + 2(2\vec{\imath} + 1\vec{\jmath}) + (1\vec{\imath} - 1\vec{\jmath})$

$\vec{a}_{(1,2,3)} = \left(21\vec{\jmath} + 14\vec{k} + 6\vec{\imath} + 2\vec{k} + 4\vec{\imath} + 2\vec{\jmath}\right) + (1\vec{\imath} - 1\vec{\jmath})$

$\vec{a}_{(1,2,3)} = \left(10\vec{\imath} + 23\vec{\jmath} + 16\vec{k}\right) + (1\vec{\imath} - 1\vec{\jmath})$

The convective acceleration component are :(10, 23, 16) m/s^2

The local acceleration components are: $(1, -1)\frac{m}{s^2}$ along x and y directions

The total acceleration of fluid particles at the point (1, 2, 3) is

$a_{1,2,3} = \sqrt{(10+1)^2 + [23 + (-1)]^2 + 16^2} = \sqrt{11^2 + 22^2 + 16^2} =$
$29.34\frac{m}{s^2}$

3.5 *Streamlines, Path Lines, Stream Tube, Streak Lines*.

3.5.1 Streamline.

At any instant can be defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point as shown in Fig. 3.1.

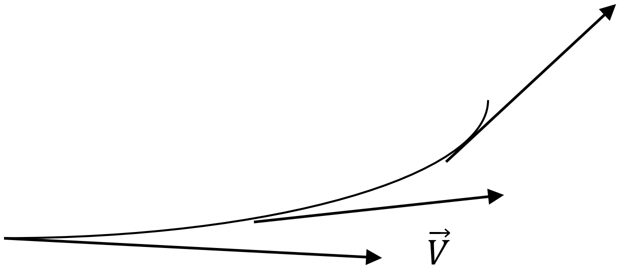


Figure 3.1: Stream line.

- In an unsteady flow where velocity vector change with time the pattern of stream lines also changes from instant to instant
- In a steady flow, the orientation as the pattern of stream line will be fixed

from above definition of stream line it can be written as

$\vec{V} \times \vec{ds} = 0$

$\vec{ds}$ The length of an infinitesimal line segment along a stream line at a point.

$\vec{V}$ The instantaneous velocity vector.

$d\vec{s} = \vec{i}dx + \vec{j}dy + \vec{k}dz \quad ; \quad \vec{V} = \vec{i}u + \vec{j}v + \vec{k}w$

$\vec{V} \times \vec{ds} = 0$

Or

$$\begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0 \quad --\rightarrow udy = vdx; \ udz = wdx; \quad vdz = wdy$$

Or $\quad \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

Ex.5

Determine the stream lines for the two-dimensional steady flow if the velocity field is given by

$$\vec{V} = \left(\frac{V_0}{l}\right)(x\vec{i} - y\vec{j})$$

V_0 & l are constant.

Sol.

$$u = \left(\frac{V_0}{l}\right)x \; ; v = -\left(\frac{V_0}{l}\right)y$$

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{\left(\frac{V_0}{l}\right)y}{\left(\frac{V_0}{l}\right)x} = -\frac{y}{x}$$

Or by integration

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

Or $lny = -lnx + constant.$

Along stream line ***xy* = *C*** ; C is constant

Different value of C, we can plot various lines in ***x-y*** plane.

Following points about stream lines are worth noting

1- A stream line cannot intersect itself nor two stream lines can cross.

2- There cannot be any movement of the fluid mass across the streamlines

3- Streamline spacing varies inversely as the velocity; converging of stream lines in any particular direction shows accelerated flow in that direction.

4- Whereas a path lines gives the path of one particular particle at successive instant of time a streamline indicates the direction of a number of particles at the same instant.

5- The series of streamlines represent the flow pattern at an instant as in Fig. 3.2.

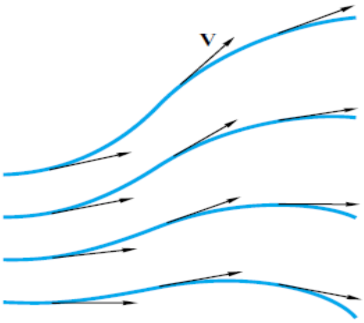


Figure 3.2: Series of streamlines.

Ex.6

Obtain the equation to the streamlines for the velocity field given as:

$\vec{V} = 2x^3\vec{i} - 6x^2y\vec{j}$

Sol.

$u = 2x^3; \;\; v = -6x^2y$

The stream line in two dimensions are defined by

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{6x^2y}{2x^3} = -\frac{3y}{x}$$

Separating the variables; we have $\frac{dy}{y} = -\frac{3dx}{x}$ By integration

$\log_e y = -3 \log_e x + c1$

Or

$\log_e y + 3 \log_e x = c1$ $\qquad yx^3 = c$

Ex.7

For a three-dimensional flow the velocity distribution is given by u = -x, v= 3-y and w = 3-z, what is the equation of a stream line passing through (1,2,2)?

Sol.

The streamlines are defined by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Substituting for u, v & w we get

$$\frac{dx}{-x} = \frac{dy}{3-y} = \frac{dz}{3-z}$$

Take the first two term; by integration

$$\int \frac{dx}{-x} = \int \frac{dy}{(3-y)}$$

$-\log_e x = -\log_e(3-y) + c1$

Where c1= constant of integration

Since the streamline passes through x=1, y=2 ∴ c1=0

$(x)^{-1} = (3-y)^{-1}\ or\ x = (3-y)$

$and \int \frac{dx}{-x} = \int \frac{dz}{3-z}$

$-\log_e x = -\log_e(3-z) + c2$

$at\ x = 1, z = 2 \quad \therefore c2 = 0$

$x^{-1} = (3-z)^{-1} \quad --\rightarrow x = (3-z)$

$$x = (3-y) = (3-z)$$

3.5.2 Path Lines.

A path line is the trajectory of fluid particle of fixed identity or a path line shows the direction of particular particle as it moves ahead as shown in Fig. 3.3.

- *One dimension flow* **:-** the single space coordinate is usually and time as flow in pipe the average values of the flow parameters are assumed
- *Two dimension flow***:-** All the flow parameters are functions of time & 2-space coordinates say (x& y)

- *Three dimensional flow*:- the parameters are function of three space coordinates and time.

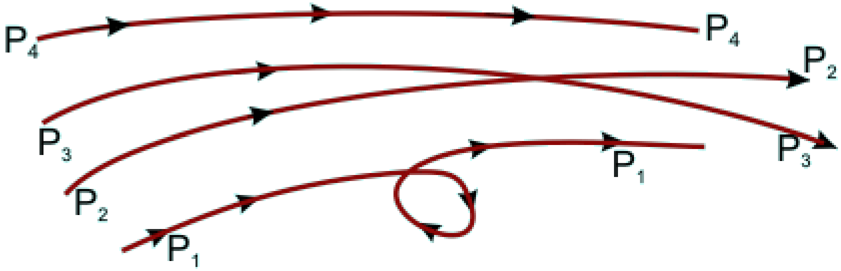


Figure 3.3: Series of path lines.

3.5.3 Stream Tube.

A fluid mass bounded by a group of stream lines. The contents of a stream tube are known as "current filament". Example, the flow as in pipes and nozzles

1- The stream tube has finite dimensions
2- As there is no flow perpendicular to stream lines therefore, there is no flow across the surface (called stream surface) as shown in Fig. 3.4.
3- The shape of a stream tube change from one instant to another, because of change is the position of streamlines.

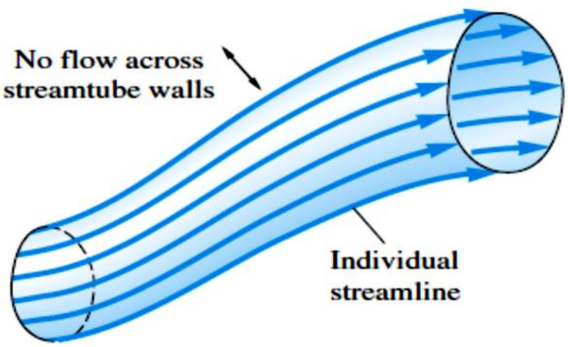


Figure 3.4: Stream tube is formed by closed collection of streamlines.

3.5.4 Streak Lines.

Streak lines provide an instantaneous picture of the particles, which have passed through a given point like the injection point of a dye in a flow.

In steady flow these lines will be coincide with stream lines. Fig. 3.5 shows the path line and streak line. Particles P_1, P_2, P_3, P_4, starting from point P at successive times pass along path lines shown. At the instant of time considered the positions of the particles are at 1, 2, 3, and 4. A line joining these points is the streak line.

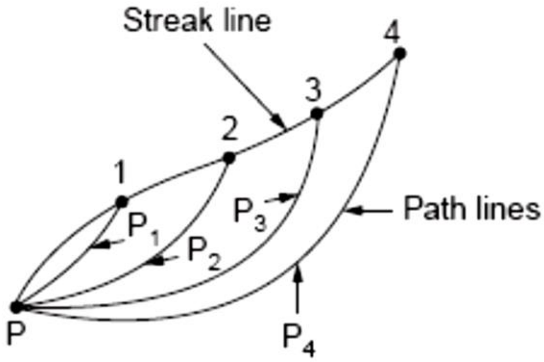


Figure 3.5: Path lines and streak lines [3].

3.6 Movement of Fluid Element.

The movement of fluid element has three distinct features in space simultaneously.

- Translation
- Rate of deformation
- Rotation

3.6.1 Pure Translation.

In Fig 3.6, the fluid element in pure translation this occur in the uniform flow field. In absence of deformation and rotation,

a) There will be no change in the length of the sides of the fluid element.

b) There will be no change in the included angles made by the sides of the fluid element.

c) The sides are displaced in parallel direction.

This is possible when the flow velocities u (the x component velocity) and v (the y component velocity) are neither a function of x nor of y, i.e., the flow field is totally uniform.

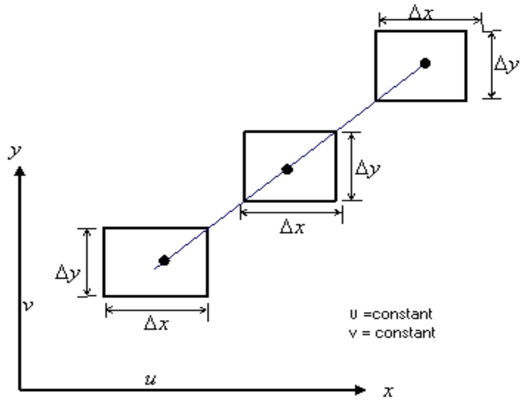


Figure 3.6: Fluid element in pure translation.

3.6.2 Linear Deformation.

If a component of flow velocity becomes the function of only one space coordinate along which that velocity component is defined. For example,

- if $u = u(x)$ and $v = v(y)$, the fluid element ABCD suffers a change in its linear dimensions along with translation
- there is no change in the included angle by the sides as shown in Fig. 3.7

The relative displacement of point B with respect to point A per unit time in x direction is $\frac{\partial u}{\partial x}\Delta x$

Similarly, the relative displacement of D with respect to A per unit time in y direction is $\frac{\partial v}{\partial y}\Delta y$

Hence, the sides move parallel from this initial position and without changing the included angle. This situation is referred to as translation with linear deformation.

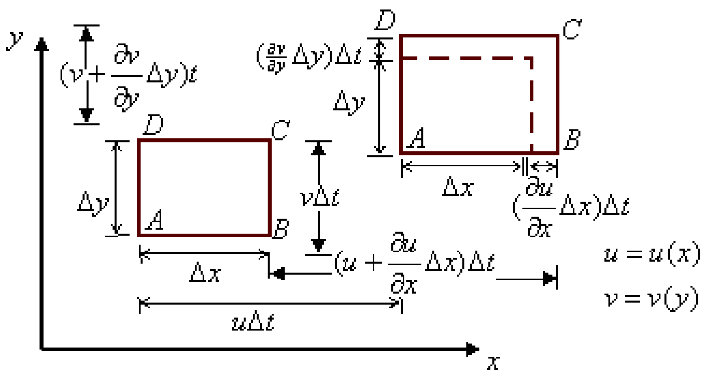


Figure 3.7: Fluid element in translation with continuous linear deformation.

Observations from Fig. 3.7 gives:

- Since u is not a function of y and *v* is not a function of x.
- All points on the linear element AD move with same velocity in the x direction.
- All points on the linear element AB move with the same velocity in y direction.
- Hence the sides move parallel from their initial position without changing the included angle.

This situation is referred to as translation with linear deformation.

Strain rate: - The changes in lengths along the coordinate axes per unit time per unit original lengths are defined as the components of linear deformation or strain rate in the respective directions.

$\in_{xx}= \frac{\partial u}{\partial x}$ Linear strain rate component in the x direction.

$\in_{yy}= \frac{\partial v}{\partial y}$ Linear strain rate component in y direction.

3.6.3 Rate of Deformation in the Fluid Element.

Let us consider both the velocity component ***u*** and ***v*** are functions of ***x*** and ***y***, i.e., *u = u(x,y)* & *v = v(x,y)*. Fig. 3.8 represents the above conditions, observations from the figure:

- Point B has a relative displacement in y direction with respect to the point A.
- Point D has a relative displacement in x direction with respect to point A.
- The included angle between AB and AD changes.
- The fluid element suffers a continuous angular deformation along with the linear deformations in course of its motion.

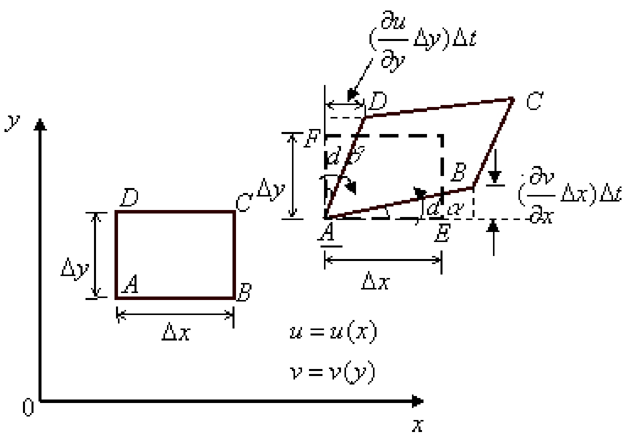


Figure 3.8: Fluid element in translation with simultaneous linear and angular deformation rates.

Rate of Angular deformation: - is defined as the rate of change of angle between the linear segments AB & AD which were initially perpendicular to each other.

The rate of angular deformation is

$\dot{\gamma}_{xy} = \left(\frac{d\propto}{dt} + \frac{d\beta}{dt} \right)$ From geometry

Hence $\frac{d\propto}{dt} + \frac{d\beta}{dt} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$

Finally

$\dot{\gamma}_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$

3.6.4 Rotation.

The transverse displacement of B with respect to A & lateral displacement of D with respect to A as in Fig. 3.8 is called the rotation of AB & AD about A. The rotation at a point is defined as the arithmetic mean of the angular velocity of two perpendicular linear segments meeting at that point. The angular velocities of AB & AD about A are $\frac{d\propto}{dt}$ & $\frac{d\beta}{dt}$ $repectively$

Considering the anticlockwise direction is positive, the rotation at A can be written as,

$$\omega_z = \frac{1}{2} \left(\frac{d\propto}{dt} - \frac{d\beta}{dt} \right) \qquad (3.9)$$

From Fig. 3.9, $d\alpha$ and $d\beta$ are each directly related to velocity derivatives in the limit of small *dt*

$$d\alpha = \lim_{dt \to 0} \left[\tan^{-1} \frac{(\partial v/\partial x)dxdt}{dx + ((\partial u/\partial x)dxdt)} \right] = \frac{\partial v}{\partial x} dt \qquad (3.10)$$

$$d\beta = \lim_{dt \to 0} \left[\tan^{-1} \frac{(\partial u/\partial y)dydt}{dy + ((\partial v/\partial y)dydt)} \right] = \frac{\partial u}{\partial y} dt \qquad (3.11)$$

Combining Eq's. (3.10 and 3.11) with Eq. 3.9 obtain the following

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \qquad (3.12)$$

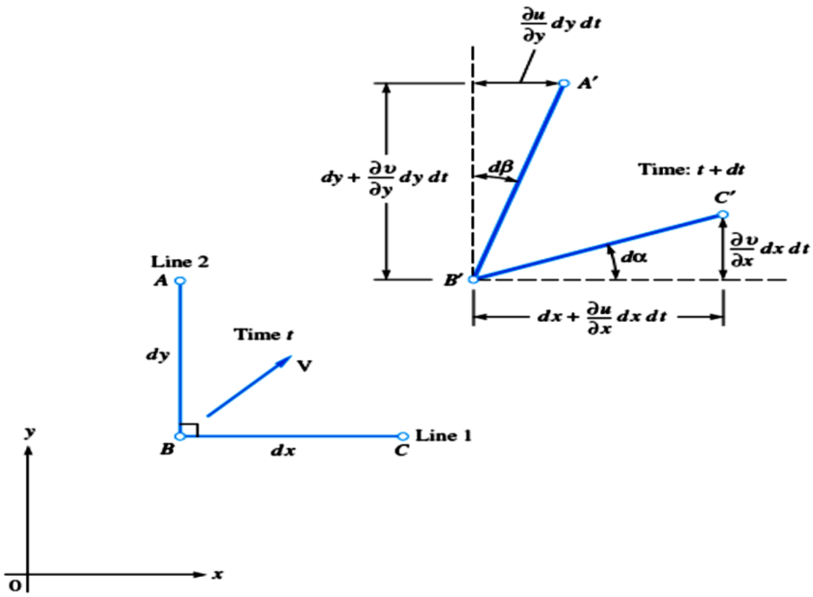


Figure 3.9: Two fluid lines deforming in xy plane.

The suffix ***z*** in ω represents the rotation about z-axis in the case of two dimensional flow along (x and y).

Rotation of $\vec{V}$, written $\frac{1}{2}(\nabla\times\vec{V})$ is curl $\vec{V}$ or rot $\vec{V}$ is defined by

$$\vec{\omega}=\frac{1}{2}\left(\nabla\times\vec{V}\right)=\frac{1}{2}\left[\left(\frac{\partial}{\partial x}\vec{\imath}+\frac{\partial}{\partial y}\vec{\jmath}+\frac{\partial}{\partial z}\vec{k}\right)\times\left(u\vec{\imath}+v\vec{\jmath}+w\vec{k}\right)\right]$$

$$\vec{\omega}=\frac{1}{2}\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ u & v & w\end{vmatrix}=\frac{1}{2}\begin{vmatrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ v & w\end{vmatrix}\vec{\imath}-\frac{1}{2}\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial z}\\ u & w\end{vmatrix}\vec{\jmath}+\frac{1}{2}\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\\ u & v\end{vmatrix}\vec{k}\quad(3.13)$$

$$\vec{\omega}=\frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\vec{\imath}+\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\vec{\jmath}+\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\vec{k}$$

$$\vec{\omega}=\frac{1}{2}\left(\omega_x\,\vec{\imath}+\omega_y\vec{\jmath}+\ \omega_z\vec{k}\ \right)\quad(3.14)$$

For three-dimensional flow the rotation is possible about three-axes. The expression for rotation ω_x, ω_y and ω_z can be obtained in like manner,

$$\omega_x=\frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\quad(3.15)$$

$$\omega_y=\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\quad(3.16)$$

$$\omega_z=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\quad(3.17)$$

In the vector notation, the above equation can be rewritten as

$$\vec{\omega} = \frac{1}{2}\left(\omega_x \vec{\imath} + \omega_y \vec{\jmath} + \omega_z \vec{k}\right) = \frac{1}{2}\left(\nabla \times \vec{V}\right) \qquad (3.18)$$

The vector $\left(\nabla \times \vec{V}\right)$ is the curl of velocity vector. The motion is described as irrotational when the components of rotation are zero.

For irrotational flow, the angle of rotation of the axes towards each other or away from each other should be equal *i.e.*, the condition to be satisfied for irrotational flow is,

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \quad \text{Or} \quad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad \text{for irrotational flow.} \qquad (3.19)$$

Ex.8

Verify whether the following flow fields are rotational. If so, determine the components of rotation about the coordinate axes,

(i) $u = xyz,\ v = xz,\ w = \frac{1}{2}yz^2 - xy$

(ii) $u = xy,\ v = \frac{1}{2}(x^2 - y^2)$

Sol. *(i)*

$$\frac{\partial u}{\partial y} = xz; \quad \frac{\partial v}{\partial x} = z; \quad \frac{\partial w}{\partial x} = -y; \quad \frac{\partial w}{\partial y} = \frac{1}{2} z^2 - x$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(z - xz) = \frac{z}{2}(1 - x)$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{dy} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left[\left(\frac{1}{2}z^2 - x\right) - x\right] = \frac{1}{2}\left(\frac{1}{2}z^2 - 2x\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}(yx + y) = \frac{y}{2}(x + 1)$$

(ii) $u = xy;\ v = \frac{1}{2}(x^2 - y^2);\ \frac{\partial u}{\partial y} = x;\ \frac{\partial v}{\partial x} = \frac{2}{2}x = x$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(x - x) = 0 \quad \text{Irrotational Flow}$$

Ex.9

Given that

$u = -4ax(x^2 - 3y^2)$

$v = 4ay(3x^2 - y^2)$

Examine whether these velocity components represent a physically possible two-dimensional flow, if so whether the flow is rotational or irrotational?

Sol.

Given u ,v is x, y components

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}[-4ax(x^2 - 3y^2)] = \frac{\partial}{\partial y}(-4ax^3 + 12axy^2) = 24axy$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial y}[4ay(3x^2 - y^2)] = \frac{\partial}{\partial x}(12ayx^2 - 4ay^3] = 24ayx$$

$$\omega_z = \frac{1}{2}(24ayx - 24ayx) = 0, \textit{henc the flow is irrotational.}$$

Ex.10

Examine whether the flow is rotational or irrotational at the point (1,-1,1) for the following velocity field

$\left(\vec{V} = xz^3\vec{i} - 2x^2yz\vec{j} + 2yz^4\vec{k}\right)$

Sol.

$$\vec{\omega} = \frac{1}{2}\left(\omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k}\right) = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}\right]$$

$$\vec{\omega} = \frac{1}{2}\left[\frac{\partial}{\partial y}(2yz^4) - \frac{\partial}{\partial z}(-2xy^2z)\right]\vec{i} + \left[\frac{\partial}{\partial z}(xz^3) - \frac{\partial}{\partial x}(2yz^4)\right]\vec{j} + \left[\frac{\partial}{\partial x}(-2x^2yz) - \frac{\partial}{\partial y}(xz^3)\right]\vec{k}$$

$$\vec{\omega} = (z^4 + (x^2y)\vec{i} + \left(\frac{3}{2}xz^2 - 0\right)\vec{j} + (-2xyz - 0)\vec{k}$$

$$\vec{\omega} = \frac{3}{2}\vec{j} + 2\vec{k} \qquad at\ (1,-1,1)$$

3.7 *Concepts of Circulation and Vorticity*.

Consider the closed path in a flow field as shown in Fig. 3.10.a, circulation is defined as the line integral of velocity about this closed path. The symbol used is Γ .

$$\Gamma = \oint_L V\cos\beta ds = \oint_L V\cos\beta dL$$

Where dL is the length on the closed curve, $V\cos\beta$ is the velocity component along the closed curve and β is the angle which streamlines makes with curve. The closed path may cut across several stream lines and at each point the direction of the velocity is obtained from the stream line, as tangent at that point.

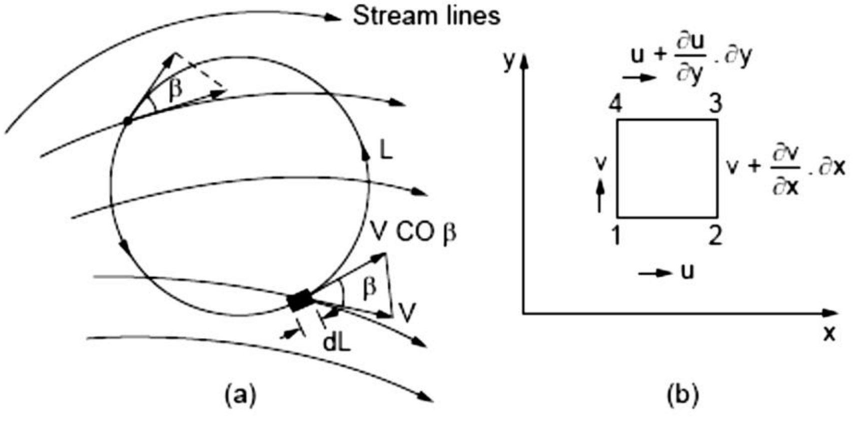


Figure 3.10: Circulations in flow.

As in Fig. 3.10.b the integration over an element can be performed. The circulation for an element *dx*,*dy* in the cartesian coordinate can be calculated as follows:

Consider the fluid element 1234 in Fig. 3.10.b starting at 1 and proceeding counter clockwise,

$$d\Gamma = udx + [v + (\partial v/\partial x)dx]dy - [u + (\partial u/\partial y)dy]dx - vdy$$

$$d\Gamma = [\partial v/\partial x - \partial u/\partial y]dxdy \qquad (3.20)$$

Vorticity is defined as circulation per unit area. *i.e.*,

$$\text{Vorticity}(\Omega) = \frac{\Gamma}{A} = \frac{d\Gamma}{dx.dy} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad (3.21)$$

Rotation (ω) is defined as one-half of the vorticity. If a flow possesses vorticity, it's rotational. For irrotational flow, vorticity and circulation are both zero.

Problems.

P3.1 The velocity of flow field is given by,

$\vec{V} = (5z - 3)i + (x + 4)j + 4yk$ ft/s, where x,y and z are in feet.

Find the fluid speed at the origin (x=y=z=0) and on the x-axis (y=z=0) **.**

P3.2 In a fluid, the velocity field is given by,

$\vec{V} = (20y/(x^2 + y^2)^{0.5})i - (20x/(x^2 + y^2)^{0.5})j$ ft/s

i- Determine the fluid speed at points along the x-axis, along y-axis.

ii- What is the angle between the velocity vector and the x-axis at point (x,y)=(5,0), (5,5) and (0,5).

P3.3 Given the velocity field,

$\vec{V} = (4 + xy + 2t)i + 6x^3j + (3xt^2 + z)k$

Find the acceleration of fluid particles,

a) As a function of (***x,y,z,t***).
b) At point (***1,1,1***) and (***t=1s***).

P3.4 Determine the stream lines for the (***2-D***) steady flow, if the velocity field is given by $\vec{V} = \frac{V_0}{L}(x_i - y_j)$.

P3.5 Obtain the stream line equation for the velocity field,

$\vec{V} = 2x^3{}_i - 6x^2y_j$.

P3.6 For a three – dimensional flow, the velocity distribution is given by,

$(u = -x), (v = 3 - y)\ and\ (w = 3 - z)$.

What is the stream line equation passing through (***1,2,2***)?

P3.7 Find the velocity and acceleration at point (***1,2,3***) after (***1s***) for the (***3-D***) flow field

$(u = yz + t), (v = xz - t) and\ (w = xy)$.

P3.8 Verify whether the flow fields are rotational. If so, determine the component of rotation about the co-ordinate axes.

(i) $u = xyz, v = xz, and\ w = \frac{1}{2}yz^2 - xy$

(ii) $u = xy,\ v = \frac{1}{2}(x^2 - y^2)$.

P3.9 Determine the components of rotation about the various axes for the flows,

(i) $u = y^2 \ and \ v = -3x$

(ii) $u = 3xy \ and \ v = \frac{3}{2}x^2 - \frac{3}{2}y^2$

(iii) $u = 3y^3z, \ v = -y^2z^2 \ and \ w = yz^2 - \frac{y^3z^2}{2}$

P3.10 Given the velocity field $\vec{V} = (6 + 2xy + t^2)i - (xy^2 + 10t)j + 25k$

i) What is the velocity Components?

ii) What the acceleration of a particle at ***(3,0,2)*** at time ***t=1s***.

P3.11 Obtain the equation to the stream line for the velocity field given as

$\vec{V} = 2x^3i - 6x^2yj$

P3.12 Determine the components of rotation about the various axes for velocity component $u = -4ax(x^2 - 3y^2)$; $v = 4ay(3x^2 - y^2)$.

CHAPTER 4

Dynamics of Fluid Flow

4.1 Introduction.

In previous chapters the forces exerted by static fluid on stationary surfaces was discussed. In this chapter the forces exerted by moving fluid particles in the flow field is discussed. In many cases the surfaces or the cross section area cause a variations or change in the magnitude and direction of the fluid particles velocity in the flow field, the fluid particles exert a force on the surface. In opposite direction the surface exert an equal force on the fluid particles. *The force exerted by moving fluid particles on the surface is called dynamic force*.

4.2 Definitions.

System:- A quantity of matter in space which is analyzed during problem.
Surrounding:- Everything external to the system .
System Boundary:- A separation present between system and surrounding.
Classifications of the system boundary:-
Real solid boundary and imaginary boundary.
The system boundary may be further classified as:-

- Fixed boundary as control mass system.
- Moving boundary as control volume system.

The choice of boundary depends on the problem being analyzed.

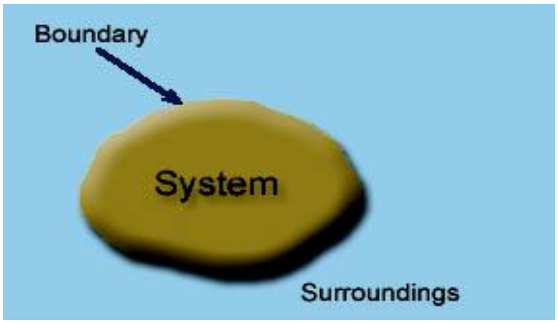
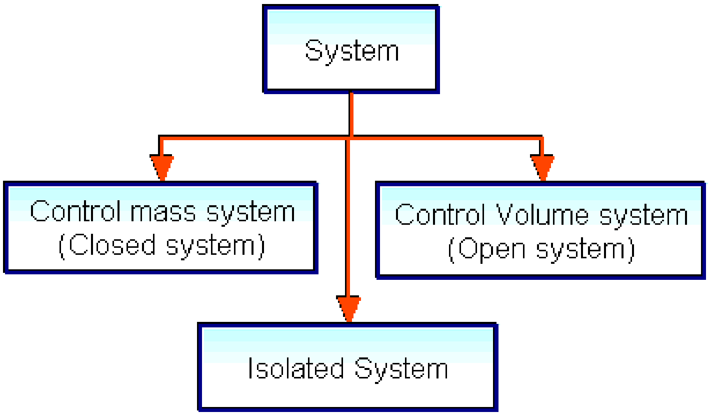


Figure 4.1: System and surroundings.

Classification of Systems.



4.3 Types of System.

a- Control Mass System (Closed System)

1. It's a system of *fixed mass* with *fixed identity*.
2. This type of system is usually referred to as *"closed system"*.
3. There is no mass transfer across the system boundary.
4. Energy transfer may take place into or out of the system as in Fig.4.2.

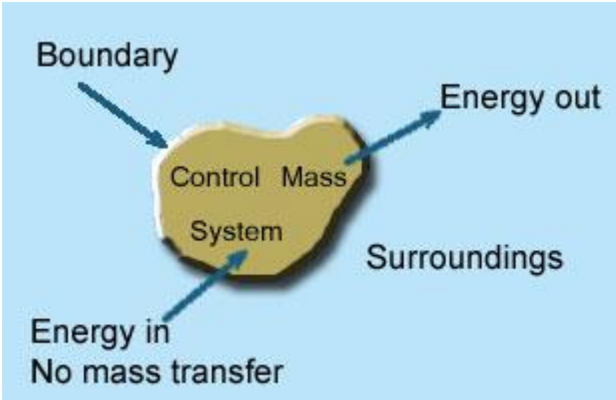


Figure 4.2: A control mass system or closed system.

b- Control Volume System (Open System)

1. It's a system of fixed *volume*.
2. This type of system is usually referred to as *"open system"* or a *"control volume" C.V.* as in Fig.4.3.
3. Mass transfer can take place across a control volume.
4. Energy transfer may also occur into or out of the system.

5. A control volume can be seen as a fixed region across which mass and energy transfers are studied.
6. Control Surface- It's the boundary of a control volume across which the transfer of both mass and energy takes place.
7. The mass of a control volume (open system) may or may not be fixed.
8. When the net influx of mass across the control surface equals zero then the mass of the system is fixed and vice-versa.
9. The identity of mass in a control volume always changes unlike the case for a control mass system (closed system).
10. Most of the engineering devices, in general, represent an open system or control volume.

Examples.

- Heat exchanger - Fluid enters and leaves the system continuously with the transfer of heat across the system boundary.
- Pump - A continuous flow of fluid takes place through the system with a transfer of mechanical energy from the surroundings to the system.

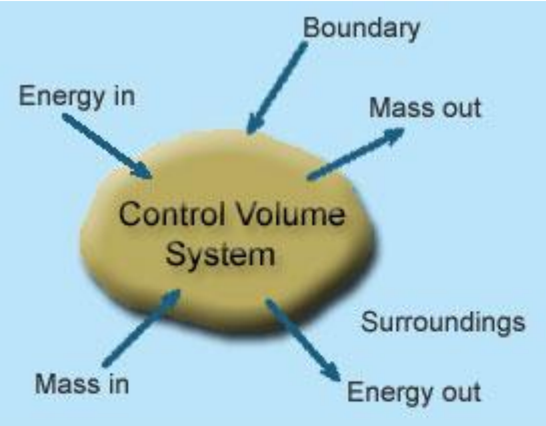


Figure 4.3: A control volume system or open system.

c- Isolated System.

1. It's a system of *fixed mass* with same *identity and fixed energy*.
2. No interaction of mass or energy takes place between the system and the surroundings as in Fig.4.4.
3. In more informal words an isolated system is like a closed shop amidst a busy market.

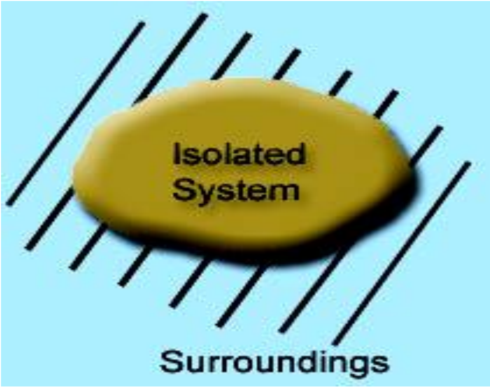


Figure 4.4: An isolated system.

4.4 Basic Laws.

1- Law of mass conservation.
2- Law of momentum conservation.
3- Law of Energy conservation.

There are two method of derivation for each law.

A- Use of differential element, and then by integration for more than one dimension.

B- Use of free body, used for one-dimension without need to integration.

4.4.1 Conservation of Mass - The Continuity Equation.

Law of conservation of mass states that *mass can neither be created nor be destroyed*. Conservation of mass is inherent to a control mass system (closed system).

- The mathematical expression for the above law is stated as:
  $\Delta m/\Delta t = 0$, where m = mass of the system
- For a control volume Fig.4.5, the principle of conservation of mass is stated as

Rate at which mass enters = Rate at which mass leaves the region + Rate of accumulation of mass in the region

Or

Rate of accumulation of mass in the control volume + Net rate of mass efflux from the control volume = 0 (4.1)

The above statement expressed analytically in terms of velocity and density field of a flow is known as the *continuity equation C.E.*

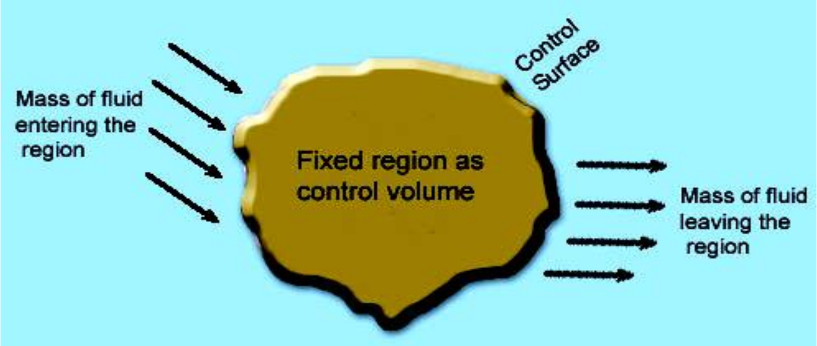


Figure 4.5: A control volume in a flow field.

4.4.2 Continuity Equation - Differential Form.

1. The point at which the continuity equation has to be derived, is enclosed by an elementary control volume.
2. The influx, efflux and the rate of accumulation of mass is calculated across each surface within the control volume.

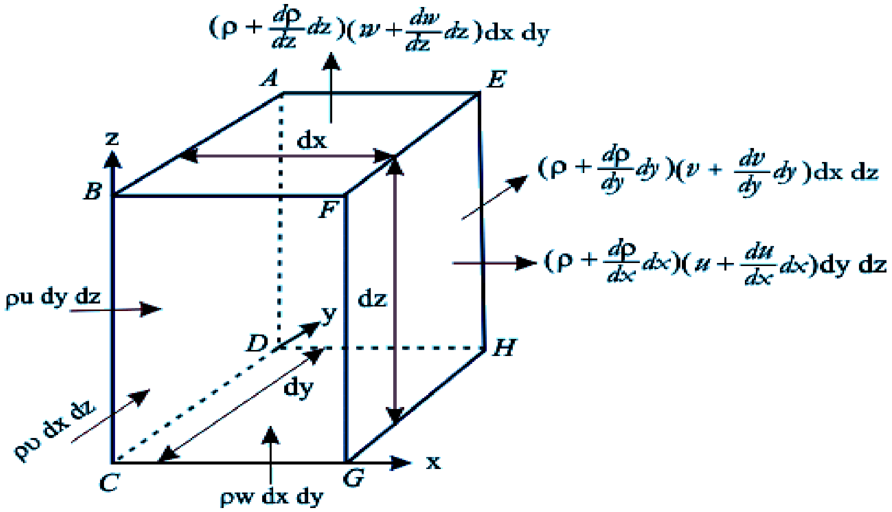


Figure 4.6: A Control volume appropriate to a rectangular cartesian coordinate system.

Consider a rectangular parallelopiped in the above figure as the control volume in a rectangular cartesian frame of coordinate axes.

- Net efflux of mass along x -axis must be the excess outflow over inflow across faces normal to x -axis.
- Let the fluid enter across one of such faces ABCD with a velocity u and a density ρ. The velocity and density with which the fluid will leave the face EFGH will be (neglecting the higher order terms in δx).

$$u+\frac{\partial u}{\partial x}\,dx \;\&\; \rho+\frac{\partial \rho}{\partial x}\,dx$$

The rate of mass entering the C.V through ABCD= ρu dy dz (a)

Therefore the rate of mass leaving the face EFGH will be

$$=\left(\rho+\frac{\partial \rho}{\partial x}\,dx\right)\left(u+\frac{\partial u}{\partial x}dx\right)dydz$$

$$=\left[\rho u+\rho\frac{\partial u}{\partial x}\,dx+u\frac{\partial \rho}{\partial x}\,dx+\frac{\partial \rho}{\partial x}\,\frac{\partial u}{\partial x}(dx)^2\right]dydz$$

Neglecting the higher order terms in (dx)

$$=\left(\rho u+\frac{\partial}{\partial x}\,(\rho u)dx\right)dydz \qquad (a')$$

Similarly influx and efflux take place in all y and z directions also.

The rate of mass entering the C.V through (BCGF) =ρv dx dz (b)

The velocity & density when the fluid leaves the face (AEHD) will be

$\left(v+\frac{\partial v}{dy}dy\right)$ & $\left(\rho+\frac{\partial \rho}{\partial y}\,dy\right)$ {Neglecting higher order} therefore rate of mass leaving the face (AEHD) will be

$$=\left[\left(\rho+\frac{\partial \rho}{\partial y}\,dy\right)\left(v+\frac{\partial v}{\partial y}\,dy\right)\right]dx\,dz$$

$$=\left[\rho v+\rho\frac{\partial v}{y}\,dy+v\frac{\partial \rho}{\partial y}\,dy+\frac{\partial \rho}{\partial y}*\frac{\partial v}{\partial y}(dy)^2\right]\,dx\,dz$$

Neglecting the higher order terms in (dy)

$$=\left[\rho v+\frac{\partial}{\partial y}\,(\rho v)dy\right]dx\,dz \qquad (b')$$

The rate of mass entering the C.V through (CDHG)= ρw dy dx (c)

The velocity & density when the fluid leaves the face (ABFE) will be

$$(\mathrm{w}+\frac{\partial \mathrm{w}}{\partial \mathrm{z}}\mathrm{dx}) \;\&\; \left(\rho+\frac{\partial \rho}{\partial \mathrm{z}}\mathrm{dz}\right)$$

Therefore the rate of mass leaving the face (ABFE) will be

$$=\left[\left(\rho+\frac{\partial \rho}{\partial z}\,dz\right)\left(w+\frac{\partial w}{\partial z}\,dz\right)\right]dy\,dx$$

$$=\left[\rho w+\rho\frac{\partial w}{\partial z}\,dz+w\frac{\partial \rho}{\partial z}\,dz+\frac{\partial \rho}{\partial z}\frac{\partial w}{\partial z}\,(dz)^2\right]\,dy\,dx$$

Neglecting the higher order terms in (dz)

$$=\left[\rho w+\frac{\partial}{\partial z}\,(\rho w)dz\right]dy\,dx \qquad (c')$$

Rate of accumulation for a point in a flow field

$$\frac{\partial m}{\partial t}=\frac{\partial \rho}{\partial t}\,d\forall \qquad (d)$$

Rate of Entering fluid = Rate of Accumulation fluid + Rate of leaving fluid

$$Eq.(a)+Eq.(b)+Eq.(c)=Eq.(d)+\;Eq(\acute{a})+Eq(\acute{b})+Eq(\acute{c})$$

$$\rho u\, dy\, dz + \rho v\, dx\, dz + \rho w\, dx\, dy = \frac{\partial \rho}{\partial t}\, d\forall + \rho u\, dy\, dz + \frac{\partial}{\partial x}(\rho u) dx\, dy\, dz + \rho v\, dx\, dz + \frac{\partial}{\partial y}(\rho v) dxdy\, dz + \rho w\, dx\, dy + \frac{\partial}{\partial z}(\rho w) dx\, dy\, dz$$

$d\forall = dxdydz,$ Rearrangement the above equation

$$\left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] d\forall = 0 \qquad (4.2)$$

This is the equation of continuity for a compressible fluid in a rectangular cartesian coordinate. The continuity equation for cylindrical polar coordinate system for a compressible fluid can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho V_r) + \frac{\rho V_r}{r} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial Z}(\rho V_z) = 0 \qquad (4.3)$$

4.4.3 Continuity Equation (C.E) - Vector Form.

If $\vec{V} = u\vec{\imath} + v\vec{\jmath} + w\vec{k}$ is the velocity of the point

$$\nabla = \left(\frac{\partial}{\partial x}\vec{\imath} + \frac{\partial}{\partial y}\vec{\jmath} + \frac{\partial}{\partial z}\vec{k}\right)$$

$$\therefore \frac{\partial \rho}{\partial t} + \left(\frac{\partial}{\partial x}\vec{\imath} + \frac{\partial}{\partial y}\vec{\jmath} + \frac{\partial}{\partial z}\vec{k}\right).\left[\rho u\vec{\imath} + \rho v\vec{\jmath} + \rho w\vec{k}\right] = 0$$

Or $\frac{\partial \rho}{\partial t} + \nabla.\left(\rho\vec{V}\right) = 0$ (4.4)

In case of a steady flow; $\frac{\partial \rho}{\partial t} = 0$

Hence Eq. (4.4) becomes

$\nabla.\left(\rho\vec{V}\right) = 0$ in a rectangelar cartesian system. (4.5)

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (4.6)$$

Eq. (4.5&4.6) represents (C.E) for a steady flow, in case of incompressible flow, ρ= constant

$\nabla.\left(\rho\vec{V}\right) = \rho\nabla.\left(\vec{V}\right)$ is the (C. E) for an incompressible fluid

$\therefore \rho\, \nabla.\left(\vec{V}\right) = 0 \quad \rightarrow \quad \nabla.\left(\vec{V}\right) = 0$ (4.7)

$or \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (4.8)

Eq. 4.8 can be written in terms of the strain rate components as

$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0$ (4.9)

The left side of the Eq's. (4.8&4.9) can be physically identified as the rate of volumetric dilatation per unit volume of fluid element in motion is obviously zero for incompressible flow.

Ex.1

The velocity distribution for a three –dimensional incompressible steady state flow is given by

$$u = 2x^2 - xy + z^2,\ v = x^2 - 4xy + y^2;\ w = -2xy - yz + y^2$$

Show that it satisfies C.E in 3-dimensions.

Sol.

in 3-D C.E is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad since\,\frac{\partial \rho}{\partial t}=0, \rho = const\ .$$

$$\frac{\partial u}{\partial x}=4x-y\ ;\ \frac{\partial v}{\partial y}=\ -4x+2y\ ;\ \frac{\partial w}{\partial z}=-y$$

Substituting in C.E

$$4x-y+(-4x+2y)-y=0$$

$$4x-y-4x+y=0 \quad statisfing\ C.E\ .$$

Ex.2

Derive the continuity equation in cylindrical polar coordinate system Eq. 4.3.

Sol.

From Fig.4.6 the rate of mass entering the control volume through face ABCD=$\rho V_r\, rd\theta dz$, and the rate of mass leaving the C.V through the face

$$EFGH=\rho V_r rd\theta\, dz+\frac{\partial}{\partial r}\,(\rho V_r\, rd\theta dz)dr$$

Therefore the net rate of mass efflux in the r-direction

$$=\rho V_r\, rd\theta\, dz+\frac{\partial}{\partial r}\,(\rho V_r\, rd\theta\, dz)dr-\rho V_r\, rd\theta\, dz$$

$$=\frac{\partial}{\partial r}\,(\rho V_r\, rd\theta\, dz\, dr)=\frac{\partial}{\partial r}\,(\rho V_r)d\forall\ =\ \frac{1}{r}\frac{\partial}{\partial r}\,(\rho r V_r)d\forall$$

$$d\forall = rdr\ d\theta\ dz$$

The net rate of mass efflux from C.V in θ direction = (mass leaving through face ADHE- mass entering through face BCGF)

$$=\rho V_\theta dr\ dz+\frac{\partial}{\partial \theta}\,(\rho V_\theta\ dr\ dz)d\theta-\rho V_\theta\ drdz=$$

$$\frac{1\partial}{r\partial\theta}\,(\rho V_\theta rdrd\theta dz)\quad =\frac{1\partial}{r\partial\theta}\,(\rho V_\theta)d\forall$$

The net rate of mass efflux in z-direction by similar fashion

$$=\rho V_z\ dr(rd\theta)+\frac{\partial}{\partial z}\,(\rho V_z\, rd\theta\ dr)dz-\rho V_z\ dr\ (rd\theta)$$

$$=\frac{\partial}{\partial z}\,(\rho V_z\, rd\theta drdz)=\frac{\partial}{\partial z}\,(\rho V_z)d\forall$$

The rate of increase of mass within the C.V becomes

$$=\frac{\partial}{\partial z}\,(\rho V_z)\ d\forall+\frac{1}{r}\frac{\partial}{\partial r}\,(\rho V_r\, r)d\forall+\frac{1}{r}\frac{\partial}{\partial\theta}\,(\rho V_\theta)d\forall$$

Hence, the fixed form of C.E in a cylindrical polar coordinate system becomes per unit volume

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\,(\rho V_r\, r)+\frac{1}{r}\frac{\partial}{\partial\theta}\,(\rho V_\theta)+\frac{\partial}{\partial z}\,(\rho V_z)=0$$

$$or\quad \frac{\partial\rho}{\partial t}+\frac{\partial}{\partial r}\,(\rho V_r)+\frac{\rho V_r}{r}+\frac{1}{r}\frac{\partial}{\partial\theta}\,(\rho V_\theta)+\frac{\partial}{\partial Z}\,(\rho V_z)=0$$

In case of an incompressible flow.

$$\frac{\partial V_r}{\partial r}+\frac{V_r}{r}+\frac{1}{r}\frac{\partial V_\theta}{\partial\theta}+\frac{\partial V_z}{\partial z}=0$$

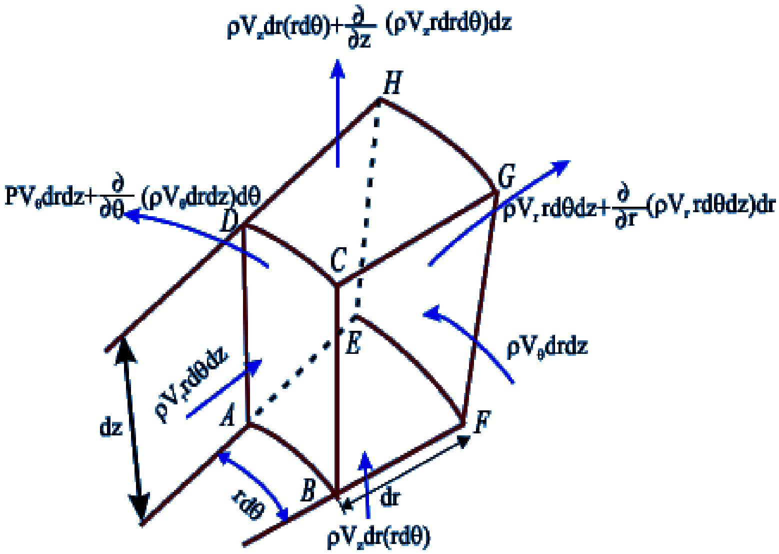


Figure 4.6: A control volume appropriate to a cylindrical polar coordinate system.

4.4.4 Free Body Method.

From Fig. 4.7 the fluid at line KL moves to new position K'L' in time Δt form the mass conservation law, the mass in (KK') equal to mass in (LL'), then

$\rho_1 A_1\, ds_1 = \rho_2 A_2\, ds_2$ (b1)

Divided the Eq. b1 by Δt

$\rho_1 A_1\, V_1 = \rho_2 A_2\, V_2$ (b2)

(V_1&V_2) represents the average velocity in cross-section *1&2*, A_1&A_1 represents the area of cross-section of the pipe in 1&2. Eq. (b2) represents the mass per time (kg/s)

$\dot{m} = \rho A V = const.$ (b3)

If the fluid is incompressible ρ=const.

$A_1\, V_1 = A_2\, V_2$ (b4)

The rate of flow as discharge, is defined as the quantity of a liquid flowing per second through a section of pipe or a channel and it's denoted by Q.

∴Discharge, Q= A*V = (m^3/s)

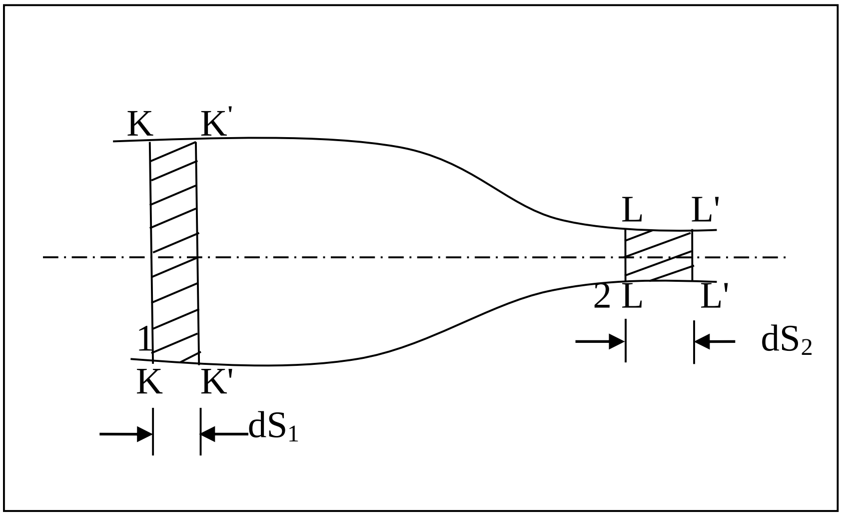


Figure 4.7: Free body diagram.

Ex-3.

As in Fig. 4.7 the diameter at cross-section (1) is equal to (12 cm), the diameter at cross-section (2) is equal to (8 cm). If the velocity at section (1) is 1.5 m/s, calculate the velocity at section (2)

Sol.

the cross-section area at (1) is

$A_1 = \frac{\pi d_1^2}{4} = \pi \frac{(0.12)^2}{4} = 0.0113\, m^2$

$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi (0.08)^2}{4} = 5.026 * 10^{-3} m^2$

$A_1\, V_1 = A_2\, V_2$

$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0113*1.5}{5.026*10^{-3}} = 3.375\, m/s$

4.5 Energy Equation of an Ideal Flow along a Stream Line.

Derivation of Bernoulli's Equation.

Euler's equation along a streamline is derived by applying Newton's second law of motion to fluid element moving along a stream line.

Considering gravity as the only the body force component acting vertically downward, the net external force acting on the fluid element moving along the direction of stream line as shown in Fig. 4.8, the equation of motion given as

$\sum F_s = m a_s$ (4.10)

Take the velocity function of s & t , V(s , t)

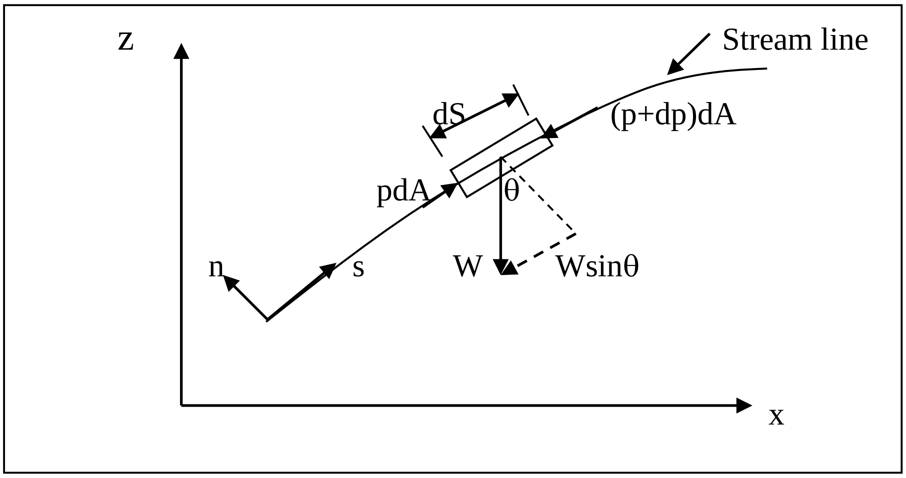


Figure 4.8: Fluid element moving along stream line.

Total differential of V(s,t)is

$dV = \frac{\partial V}{\partial s}\, ds + \frac{\partial V}{\partial t}\, dt$ Divided by dt

$\frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$

In steady flow $\frac{\partial V}{\partial t} = 0$

Then V=V(s) , the acceleration in the s-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s}\, V = V\frac{dV}{ds} \qquad (4.11)$$

The forces acting in the s-direction are the pressure and the component of particle weight in the s-direction from Eq's. (4.10 &4.11)

$$p\, dA - (p + dp)dA - W sin\theta = m\, V\frac{dV}{ds} \qquad (4.12)$$

Where θ is the angle between the normal to the streamline and the vertical z-axis at that point.

$m = \rho\forall = \rho\, dA\, ds$

$W = mg = \rho g\, dAds$

$sin\theta = \frac{dz}{ds}$

Substituting in Eq. 4.12

$$-dp\, dA - \rho g\, dA\, ds\frac{dz}{ds} = \rho\, dA\, ds\, V\frac{dV}{ds} \qquad (4.13)$$

Divided by dA and simplifying

$-dp - \rho g\, dz = \rho V\, dV$

Note V dV = $\frac{1}{2}$ d(V^2)& dividing each term by ρ gives

$$\frac{dp}{\rho} + \frac{1}{2}\, d(V^2) + g\, dz = 0 \qquad (4.14)$$

Integrating along streamline

$$\int \left(\frac{dp}{\rho} + \frac{1}{2}\, d(V^2) + \, g\, dz\right) = constant$$

For, incompressible ρ= const.

$$\frac{p}{\rho} + \frac{V^2}{2} + \, gz = constant$$

This is the famous Bernoulli's equation *B.E* used for steady incompressible & inviscid regions flow. The *B.E* can be written between any two points on the same streamline as

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \, gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \, gz_2 \qquad (4.15)$$

Eq. 4.15 can be written as

$$\frac{p}{\rho} + \frac{V^2}{2} + \, gz = C \qquad (4.16)$$

Where *C* is a constant along a streamline. In case of an incompressible flow, Eq. 4.16 is based on the assumption no work or heat interaction between a fluid element and the surrounding take place, its terms illustrate as follows,

- 1st term represents the flow work per unit mass.
- 2nd term represents the kinetic energy per unit mass.
- 3rd term represents the potential energy per unit mass.

The sum of three terms represents the total mechanical energy per unit mass which remains constant along a streamline for steady, invicid & incompressible flow of fluid. Eq. 4.16 is known as mechanical energy equation. Also, Eq. 4.16 can be expressed in terms of energy per unit weight as

$$\frac{p}{\rho g} + \frac{V^2}{2g} + \, z = C1 \qquad (4.17)$$

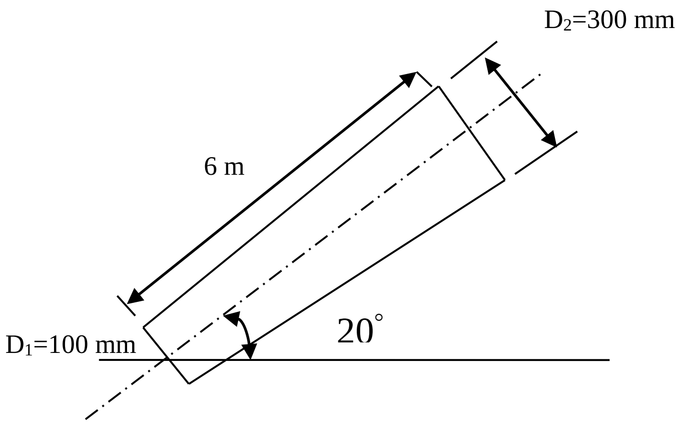
The energy per unit weight is termed as a ***Head***, Eq. 4.17 can be written as
(Pressure head)+(Velocity head)+ (Potential head)= Total head
In many practical situations, problems related to real fluid and can be analyzed with help of a modified form of Bernoulli's equation as

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \, z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \, z_2 + h_f \qquad (4.18)$$

Where $\boldsymbol{h_f}$ represents the frictional work done (the work done against the fluid friction) per unit weight of fluid element.

<u>Ex.4</u>

A 6m long pipe is inclined at angle of 20° with the horizontal. The smaller section of the pipe which is at lower level is of 100mm and the larger section of pipe is of 300 mm diameter as shown in figure. If the pipe is uniformly tapering and the velocity of water at the smaller section is 1.8 m/s determine the difference of pressures between the two sections.



Sol.

$A_1 = \frac{\pi d1^2}{4} = \pi * \frac{0.1^2}{4} = 0.00785 m^2$

$V_1 = 1.8 \frac{m}{s}$

$z_1 = 0\ m$

$d_2 = 0.3m$

$A_2 = \frac{\pi}{4} * 0.3^2 = 0.0707 m^2$

$z_2 = 6 sin 20 = 6 * 0.342 = 2.05m$

From *C.E* $A_1V_1 = A_2V_2$

$\therefore V_2 = \frac{A_1V_1}{A_2} = 0.00785 * \frac{1.8}{0.0707} = 0.2\ m/s$

Applying B.E. to both sections of pipe

$\frac{p1}{\gamma} + \frac{V1^2}{2g} + z_1 = \frac{p2}{\gamma} + \frac{V2^2}{2g} + z_2$

$p_1 - p_2 = \gamma\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$

$p_1 - p_2 = 9810\left(\frac{0.2^2 - 1.8^2}{2*9.81} + 2.05\right) = 18510 \frac{N}{m^2}$

Ex.5

a) Determine the velocity of efflux from the nozzle in the wall of the reservoir as in figure.
b) Find the discharge at the nozzle.

Sol.

a) $\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2$

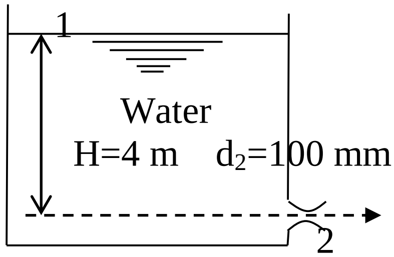
With pressure datum as local atmospheric pressure, $p_1=p_2=0$ & $z_2=0$, $z_1=H$, the velocity on the surface of the reservoir is zero Hence .

$$0 + 0 + H = \frac{V_2^2}{2g} + 0 + 0$$

$V_2 = \sqrt{2gH} = \sqrt{2 * 9.81 * 4} = 8.86 \frac{m}{s}$

This is known as torricellis theorem

b) $Q = A_2V_2 = \pi(0.05)^2(8.86) = 0.07\frac{m^3}{s} = 70\ \frac{L}{s}$



4.6 Conservation of Momentum.

In Newtonian mechanics, the conservation of momentum is defined by Newton's second law of motion. The 2nd law of motion states as

- The rate of change of momentum of a body is proportional to the impressed action and takes place in the direction of the impressed action.
- If a force acts on the body, linear momentum is implied.
- If a torque (moment) acts on the body angular momentum is implied.

Statement of Reynolds Transport Theorem, "*the time rate of increase of property (N) within a control mass system (CMS) is equal to the time rate of increase of property (N) within the control volume (CV) plus the net rate of efflux of the property (N) across the control surface* (CS)".

$$\left(\frac{dN}{dt}\right)_{CMS} = \frac{\partial}{\partial t}\iiint_{C.V}\eta\,\rho d\forall + \iint_{C.V}\eta\rho\,\vec{V_r}\,d\vec{A} \qquad (4.19)$$

Eq. 4.19 is known as Reynolds Transport Theorem, $\vec{V_r} = \vec{V} - \vec{V_c}$

$\vec{V_r}$= fluid velocity relative to *C.V.*

$\vec{V}$&$\vec{V_c}$ = velocities of fluid & *C.V.* as observed in a fixed frame reference.

N= flow property which is transported.

η= intensive value of the flow property.

4.6.1 Linear Momentum.

From Eq. 4.19, the property *N* as in the linear- momentum m$\vec{V}$ & η as the velocity $\vec{V}$. Then it becomes

$$\frac{d}{dt}\left(m\vec{V_r}\right)_{CMS} = \frac{\partial}{\partial t}\iiint_{CV}(\vec{V_r})\rho\ d\forall + \iint_{CS}\vec{V_r}\ \rho\left(\vec{V_r}.d\vec{A}\right) \qquad (4.20)$$

Where $\vec{V_r}$ is the velocity defining the linear momentum in above equation. L.H.S of Eq. 4.20 represents the external forces $\sum\vec{F}$ on the *CMS* or on the coinciding *C.V* by the direct application of Newton's law of motion. The L.H.S of Eq. 4.20 form Newton's law of motion can be written as

$$m\left(\frac{d\vec{V}}{dt}\right)_{CMS} = \sum \vec{F}$$

Therefore

$$\left(m\frac{d\vec{V}_r}{dt}\right)_{CMS} = m\left(\frac{d\vec{V}_r}{dt}\right)_{CMS} = m\frac{d}{dt}\left(\vec{V} - \vec{V}_C\right)_{CMS} = m\left(\frac{d\vec{V}}{dt}\right)_{CMS} - m\vec{a}_C$$

Where $a_c = \left(\frac{d\vec{V}_C}{dt}\right)$ is the rectilinear acceleration of the *C.V* (observed in a fixed coordinate system).

Therefore, $m\left(\frac{d\vec{V}_r}{dt}\right)_{CMS} = \sum \vec{F} - m\vec{a}_C$ (4.21)

Eq. 4.20 can be written as follows after consider Eq. 4.21

$$\sum \vec{F} - m\vec{a}_C = \frac{\partial}{\partial t} \iiint_{CV} (\vec{V}_r)\rho \; d\forall + \iint_{CS} \vec{V}_r \; \rho \left(\vec{V}_r . d\vec{A}\right) \quad (4.22)$$

At steady state form it becomes

$$\sum \vec{F} - m\vec{a}_C = \iint_{CS} \vec{V}_r \; \rho \left(\vec{V}_r . d\vec{A}\right) \quad (4.23)$$

In case of an inertial C.V, which is either fixed or moving with a constant rectilinear velocity $\vec{a}_c = 0$, Now, Eq's. (4.22 & 4.23) becomes

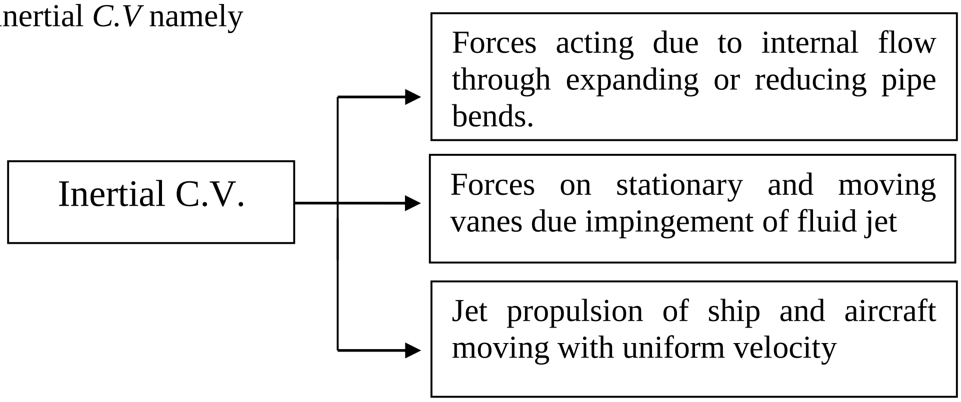
$$\sum \vec{F} = \iint_{CS} \vec{V}_r \; \rho \left(\vec{V}_r . d\vec{A}\right) \quad (4.24)$$

Or $\sum \vec{F} = \frac{\partial}{\partial t} \iiint_{CV} (\vec{V}_r)\rho \; d\forall + \iint_{CS} \vec{V}_r \; \rho \left(\vec{V}_r . d\vec{A}\right)$ (4.25)

Eq's (4.22 & 4.23) for non-inertial C.V having an arbitrary rectilinear acceleration.

4.6.2 The Application of Momentum Theorem.

From the conservation of momentum phenomenon we can state the law of conservation of momentum as follows " the net force acting on a mass of fluid is equal to change in momentum of flow per unit time in that direction". The application of momentum theorem in some practical cases of inertial and non-inertial *C.V* can be treated. Three distinct types of practical problems for inertial *C.V* namely



Non-Inertial C.V	→	Rocket engine which works on the principle of jet propulsion.

Liner momentum of *C.V* in a system is (m.$\vec{V}$) from Newton's 2nd law

$$\sum \vec{F} = \frac{d(m\vec{V})}{dt} = \frac{\partial}{\partial t}\int_{C.V} \rho\, \vec{V}\, d\forall + \int_{C.S} \rho\vec{V}\, (\vec{V}.d\vec{A})$$

(i.e) the resultant force acting on a *C.V* is equal to the time rate of increase of linear momentum within the *C.V* plus the net output of linear momentum from the *C.V*. Types of forces acting on control volume (*C.V*)

- Body force the weight of fluid
- Pressure force
- Hydrostatic force
- Shear force

$$\sum \vec{F}_{total} = \sum \vec{F}_{gravity} + \sum \vec{F}_{pressure} + \sum \vec{F}_{viscous} \qquad (4.26)$$

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}\left(m\vec{V}\right)$$

Where $(m\vec{V})$ is the linear momentum of system. Now the 2nd law can be expressed more general as

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho\, \vec{V}\, d\forall \qquad \text{where } dm = \rho d\forall$$

External force equal to the time rate of change of linear momentum of system. If system at rest or move with constant velocity then from Reynolds Transport Theorem is applied on *C.V* formulation as follows

$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{\partial}{\partial t}\iiint_{CV} \rho(\vec{V}\ )\, d\forall + \iint_{CS} \rho\ \ \vec{V}\ \left(\vec{V}_r.\vec{n}\right)dA \qquad (4.27)$$

But the left side of Eq. 4.27 is equal to $\sum \vec{F}$

$$\sum \vec{F} = \frac{\partial}{\partial t}\iiint_{CV} \rho(\vec{V}\ )\, d\forall + \iint_{CS} \rho\ \ \vec{V}\ \left(\vec{V}_r.\vec{n}\right)dA \qquad (4.28)$$

Eq. 4.28 can be expressed as

(The sum of all external force acting on C.V)= (The times rate of change of linear momentum of the contents of the C.V) + (The net flow rate of LM out of the C.V by mass flow).

Here $\vec{V}_r = \vec{V} - \vec{V}_{cs}$ is the fluid velocity relative to the *C.V*. $\vec{V}$ is the velocity of fluid as viewed from fixed reference frame. The product $\rho\left(\vec{V}_r.\vec{n}\right)dA$ represents the mass flow rate through area element dA into or out of the *C.V* for fixed *C.V*. For fixed *C.V* no motion of *C.V* or deformation $V_r = V$ and the linear-momentum equation for fixed *C.V* becomes

$$\sum \vec{F} = \frac{d}{dt}\int_{C.V} \rho\, \vec{V}\, d\forall + \int_{CS} \rho\, \vec{V}\left(\vec{V}.\vec{n}\right)dA \qquad (4.29)$$

For steady the derivative with respect to time is equal to zero

$$\sum \vec{F} = \int\int_{CS} \rho\, \vec{V}\left(\vec{V}.\vec{n}\right)dA \qquad (4.30)$$

Mass flow rate across an inlet or outlet

$$\dot{m} = \int_{Ac} \rho(\vec{V}.\vec{n})dA_c = \rho V_m A_c \tag{4.31}$$

∴Momentum flow rate across inlet or outlet

$$\int_{Ac} \rho\, \vec{V}(\vec{V}\,.\vec{n})dA_c = \rho V_m A_c\, \vec{V}_m = \dot{m}\, \vec{V}_m \tag{4.32}$$

V_m= uniform mean velocity

$$\sum \vec{F} = \sum_{out} \dot{m}\vec{V} - \sum_{in} \dot{m}\vec{V} \tag{4.33}$$

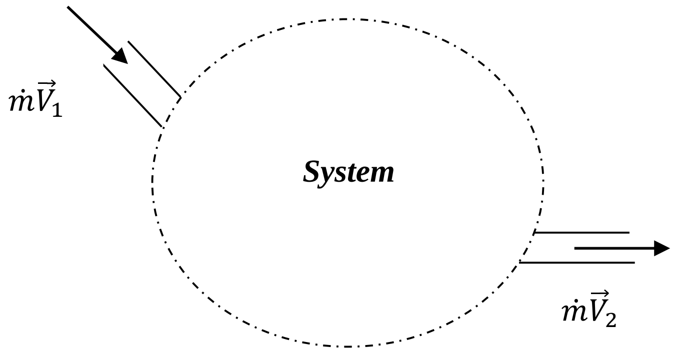


Figure 4.9: Linear momentum of system.

Along x-direction as in Fig.4.9

$\sum \vec{F}_x = \dot{m}(\vec{V}_{2x} - \vec{V}_{1x})$

Similarly $\sum F_y = \rho Q(V_{2y} - V_{1y}) = \dot{m}(V_{2y} - V_{1y})$

For any C.V the total force $\vec{F}$ which acts upon it in a given direction will made up of three components $\vec{F} = F1 + F2 + F3$

- F1=force exerted in the direction on the fluid in the *C.V* by any solid body within the C.V or coinciding with boundaries of the *C.V.*
- F2=force exerted in the given direction on the fluid in the C.V by body for such as gravity.
- F3=force exerted in the give direction of fluid in the C.V by the fluid outside the C.V such as pressure.

The effects of these forces on *C.V* can be study through practical engineering problem from the following cases.

I- Forces due to Flow through Expanding or Reducing Pipe Bends.

Fig's 4.10 and 4.11 shows the fluid flow through an expander where F_x &F_y are the external forces on the fluid areas 2-3 & 1-4 due to net efflux linear momentum through the interior surface of the expander since *C.V* (1 2 3 4) is stationary and at steady state apply Eq.4.24 for *x & y* components.

$\dot{m}V_2\, cos\theta - \dot{m}V_1 = p_1A_1 - p_2A_2\, cos\theta + F_x$

And $\quad \dot{m}V_2\, sin\theta - 0 = -p_2A_2 sin\theta + F_y - mg$

Or $F_x = \dot{m}(V_2\, cos\theta - V1) + p_2 A_2\, cos\theta - p_1 A_1$ (4.34)

$F_y = \dot{m} V_2 sin\theta + p_2 A_2\, sin\theta + mg$ (4.35)

Where

$\dot{m}$=mass flow rate through the expander, analytically it can be expressed as
$\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$

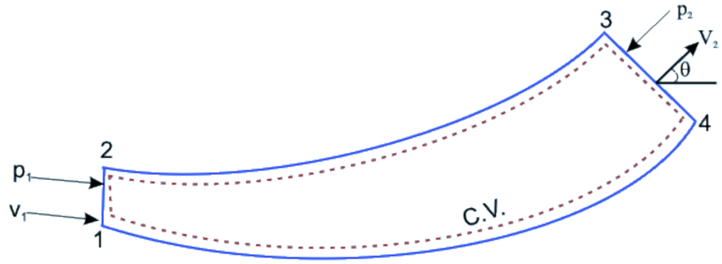


Figure 4.10: Flow of a fluid through an expander.

m represents the mass of fluid contained in the expander at any instant and can be expressed as
$m = \rho \forall$ Where $\forall$ internal volume of the expander, F_x & F_y forces acting on the $C.V$ by the expander. According to Newton's third law (any action there is a reaction) the expander will experience the forces
$R_x = -F_x$ & $R_y = -F_y$ are the reactions in the x&y directions respectively as shown in the free body diagram of the expander Fig. 4.12.

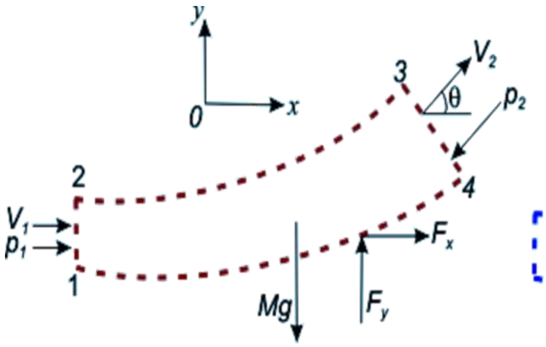


Figure 4.11: Control volume comprising the fluid contained in the expander at any instant.

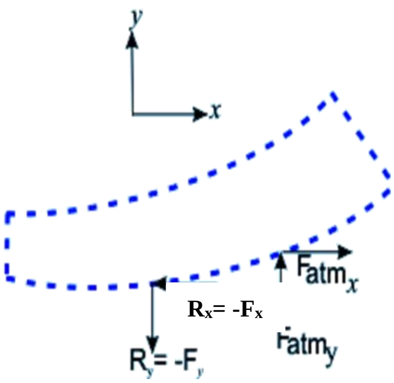


Figure 4.12: Free body diagram of the expander.

The magnitude of the resultant force acting on the pipe bend is

$R = \sqrt{R_x^2 + R_y^2}$

And the direction of the resultant force with x-axis is

$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right)$

Ex.6

A 600 mm diameter pipeline carries water under a head of 30 m with velocity of 3 m/s. This water main is fitted with a horizontal bend which turns the axis of the pipeline through 75°. Calculate the resultant force on the bend and its angle to the horizontal.

Sol.

$A_1 = A_2 = \pi\left(\frac{0.6}{2}\right)^2 = 0.283\, m^2$

$d = 0.6m, h = 30m$

$V_1 = V_2 = 3\frac{m}{s}, p_1 = \rho gh = 9810 * 30 = 294300\, {}^{N}/{}_{m^2}$

$Q = A_1V_1 = A_2V_2 = 0.283 * 3 = 0.849\, {}^{m^3}/{}_{s}$

From Eq.(4.34 & 4.35) $\dot{m} = \rho Q = 1000 * 0.849 = 849\frac{kg}{s}$

$F_x = \dot{m}(V_2\, cos\theta - V1) + p_2A_2\, cos\theta - p_1A_1$

$F_x = 849(3 * cos75 - 3) + 294300 * 0.283 * cos75 - 294300 * 0.283$

$F_x = -63.618\, kN$

$F_y = \dot{m}V_2 sin\theta + p_2A_2\, sin\theta + mg$

$= 849 * 3 * sin75 + 294300 * 0.283 * sin75$ Since the bend in horizontal

$Fy = 82.9\, kN$

$\therefore\ R_x = -F_x = +63.618\, kN,\ R_y = -F_y = -82.9kN$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(63.618)^2 + (-82.9)^2} = 104.5\, kN$

$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) = tan^{-1}\left(\frac{-82.9}{63.618}\right) = -52.5^{\circ}$

Ex.7

A pipe bend tapers from a diameter d, of (500) mm at inlet to a diameter d2 of (250mm) at outlet and turns the flow through an angle (θ) of 45°. Measurements of (p_1 & p_2) at inlet and outlet are 40 kN/m^2 and 23 kN/m^2. If the pipe is conveying oil which has a density ρ=850 kg/m^3. Calculate the magnitude and direction of resultant force on the bend when the oil is flowing at the rate of 0.45 m^3/s. The bend is in a horizontal plan. (Gravity force=0)

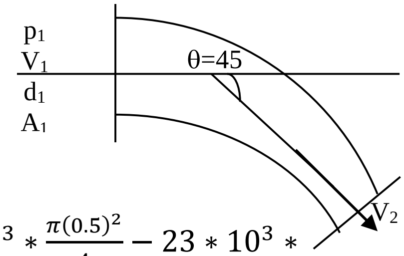
Sol.

$\dot{m} = \rho AV = \rho Q = 850 * 0.45 = 382.5\, kg/s$

$$F_x = \dot{m}(V_{2x} - V_{1x}) + p_1A_1 - p_2A_2\cos\theta$$

$$V_2 = \frac{\boldsymbol{Q}}{\boldsymbol{A_2}} = 0.45 * \frac{4}{\pi(0.25)^2} = 9.16\ \mathrm{m/s}$$

$$V_1 = \frac{Q}{A_1} = 0.45 * \frac{4}{\pi(0.5)^2} = 2.29\ \mathrm{m/s}$$



$$F_x = 382.5(9.16\cos 45 - 2.29) + 40 * 10^3 * \frac{\pi(0.5)^2}{4} - 23 * 10^3 * \frac{\pi(0.25)^2}{4}\cos 45$$

$$\therefore F_x = 8657N \quad - - \rightarrow \mathrm{R_x} = -8657\ N$$

$$F_y = \dot{m}\left(V_{2y} - V_{1y}\right) + p_2A_2\sin\theta = 382.5(9.16\sin 45 - 0) + 23 * 10^3 * \sin 45 = 18740.9\ N$$

$$R_y = -F_y = -18740.9\ N$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-8657)^2 + (-18740.9)^2} = 20643\ N$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = 65^{\circ}$$

II-Dynamic Force on Plane Surfaces due to the Impingement of Liquid Jet.

A. Forces on a Stationary Surface.

Consider a stationary flat plate and a liquid jet of cross sectional area (**a**) striking with a velocity *V* at an angle *θ* to the plate as shown in Fig. 4.13.a.

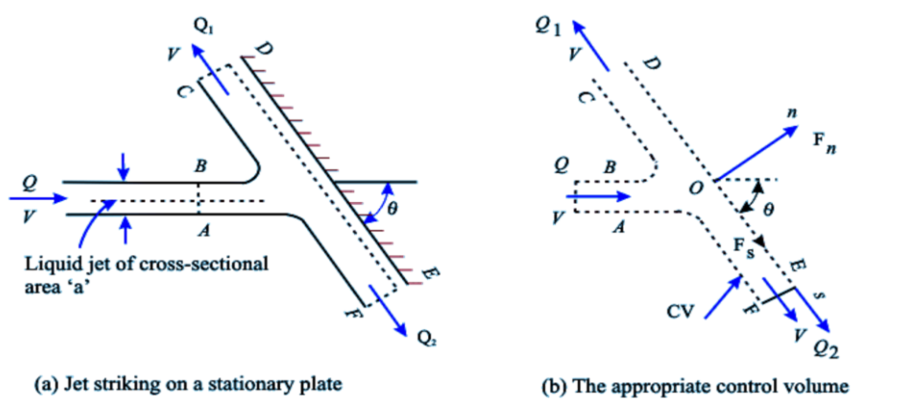


Figure 4.13: Impingement of liquid jets on a stationary flat plate.

To calculate the force required to keep the plate stationary, a control volume ABCDEFA Fig. 4.13.a is chosen so that the control surface DE coincides with the surface of the plate. The control volume is shown separately as a free body in Fig. 4.13.b. Let the volume flow rate of the incoming jet be Q and be divided into Q_1 and Q_2 gliding along the surface with the same velocity V since the pressure throughout is same as the atmospheric pressure, the plate is considered to be frictionless and the influence of a gravity is neglected (i.e. the elevation between sections CD and EF is negligible).

- Q(m^3/s) incoming jet plane flow rate
- (Q1&Q2)m^3/s gliding along the plate output volume flow rate with the same velocity
- Plate is friction less
- Gravity is neglected due to elevation between section CD & EF is negligible
- *Os & On* are the axes along and perpendicular to the plate
- Neglecting the viscous force along the plate, the plate is friction less & the pressure throughout is same as the atmospheric pressure. The force along the plate is zero
- The momentum conservation of the C.V {A B C D E F} in terms of $(s\&n)$ can be written from Eq. 4.24
- $\sum \vec{F} = \int_{CS} \rho\, \vec{V}_r\, (\vec{V}.\overrightarrow{dA})$

$F_s = 0 = (\dot{m}\vec{V})_{out} - (\dot{m}\vec{V})_{in} \rightarrow or\ (\rho\ Q\ \vec{V})_{out} - (\rho\ Q\ \vec{V})_{in} = 0$

$F_s = (\rho\ Q_2\ V + \rho\ Q_1(-V)) - \rho QV\ Cos\theta = 0$ s-direction (4.36)

& $F_n = (\rho QV)_{out} - (\rho QV)_{in} = 0 - \rho QV\ Sin\theta = -\rho QV\ Sin\theta$ (4.37)

Where F_s&F_n are the forces acting on the *C.V* along *Os& On* respectively

From continuity

$Q = Q1 + Q2$ (4.38)

From Eq. 4.36

$\rho Q\ Vcos\theta = \rho Q_2 V - \rho Q_1 V$ Divided by ρV, and from Eq. 4.38

$Q2 = (Q - Q1)$

$Q\ cos\theta = (Q - Q1) - Q1 = Q - 2Q1$

$2Q1 = Q - Qcos\theta = Q(1 - cos\theta)$

$Q1 = \frac{Q}{2}(1 - cos\theta)$

By same procedure $Q1 = (Q - Q2)$

$\therefore Q2 = \frac{Q}{2}(1 + cos\theta)$

The net force acting on the C.V due to the change in momentum of the jet by the plate is F_n along the direction (*On*) and is give by Eq. 4.37 as

$F_n = -\rho QV\ sin\theta$ According to Newton's third law the force acting on the plates $F_p = -F_n = \rho QVsin\ \theta$ (4.39)

$Q = AV$ (4.40)

$F_p = \rho AV^2 sin\theta$; where A is the cross-section area (4.41)

B. Forces on a Moving Surface.

V: is the velocity of jet striking the plate
u: is the velocity of plate

V_r = V-u (4.42)

The volume of liquid striking the plate per unit time will be

$Q = A(V - u)$ (4.43)

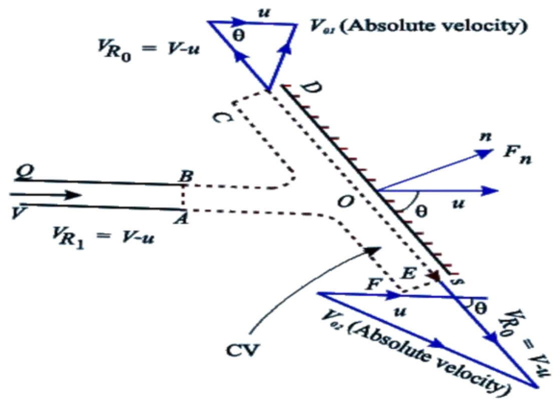


Figure 4.14: Impingement of liquid jet on a moving flat plate.

At inlet velocity of jet relative to the ***C.V*** is $V_{r1} = V - u$

- Due to the pressure remains same throughout
  $V_{R0} = V - u$ is the same as inlet
- Friction is neglected between liquid & plate.

Absolute velocity of the liquid at the outlets can be found out by adding vectorially the triangle of velocites as shown in Fig. 4.14. The force acting along (*Os*) is zero for a friction less flow, only the net force acting on the *C.V* will be along (*On*). To calculate F_n from the momentum theorem on the *C.V* can be written as

$F_n = O - \rho Q[(V - u)sin\theta]$

Substituting Q from Eq. 4.43

$F_n = -\rho A(V - u)^2 sin\theta$ (4.44)

The force acting on the plate becomes

$F_p = -F_n = \rho A(V - u)^2 sin\theta$ (4.45)

If the plate moves with velocity *u* in a direction opposite to that of *V* (the plate moving towards the jet), since Q=A (V+u)

$\therefore F_p = -F_n = \rho A(V + u)^2 sin\theta$

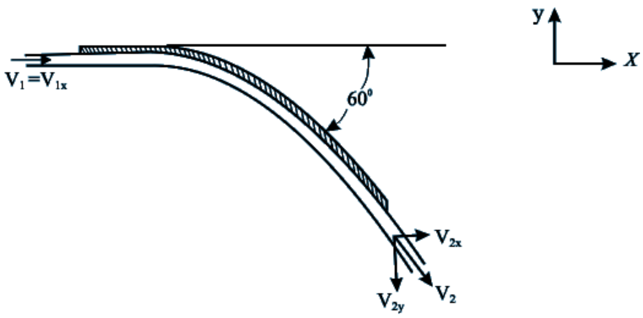
The power developed due to the motion of the plate can be written in case of the plate moving in the same direction as that of the jet as

$P = F_p * u$

$P = F_p\ sin\theta\ \ |u|{=}\rho A(V-u)^2\ u\ sin\theta$ (4.46)

Ex.8

Consider a jet that is deflected by a stationary vane, such as is given in figure. If the jet speed and diameter are 25 m/s &25 cm, respectively and the jet is deflected 60°, what force is exerted by the jet?



Sol.

First solve for $\boldsymbol{F_x}$, the x-component of force of the vane on the jet

$F_x = (\rho QV)_{x\ out} - (\rho QV)_{x\ in}$

$F_x = \rho Q(V_{2x} - V_{1x})$

$V2x = V_2\ cos60 = 25 * 0.5 = 12.5\, m/s$

$V_{1x} = 25\, m/s$

$Q = V_1A_1 = 25 * \frac{\pi(0.25)^2}{4} = 1.227\ {m^3}/{s}$

Therefore F_x = (1000)(1.227)(12.5-25)= -15.3398 kN

Similarly determined, the y-component of force on the jet is

$F_y = \rho Q\left[\left(V_y\right)_{out} - \left(V_y\right)_{in}\right] = 1000 * 1.227(-21.65 - 0) = -26.5646\ kN$

$since\ V_{2y} = Vsin60 = 25sin60 = 21.65\, m/s$

The resultant force is

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-15.3398)^2 + (-26.5646)^2} = 30.67kN$

Then the force on the vane will be the reactions to the forces of the vane on the jet as

$R_x = -F_x = +15.3398kN$

$R_y = -F_y = +26.5646kN$

C. Dynamic Forces on Curve Surfaces due to the Impingement Liquid Jets.

In determination of the force and energy transfer between the moving blades and the fluid, the relative velocity between the blade and the fluid becomes very important effective factor in calculations. The following parameters and notations will be used in our calculation of dynamic forces.

α_1 , angle with direction of motion of the vane, at which the jet enters the vane.

α_2 , angle with direction of motion at which the jet leaves the vane.

θ_1 & θ_2 , angles which V_{r1} and V_{r2} makes with direction of motion of vane.

V_1 & V_2 absolute velocities of jet at inlet & leaving the vane.

V_{r1} & V_{r2} relative velocity at entrance and exit the vane, V_r= V-u

V_{w1}&V_{w2}, horizontal components of V_1 & V_2 respectively.

V_{f1}&V_{f2}, vertical components of V_1 & V_2 respectively.

F_c is the force applied on the ***C.V*** by the vane therefore from Eq. 4.24 the momentum theorem in x-direction as,

$$F_c = \rho Q[(V_{rx})_{out} - (V_{rx})_{in}] = \dot{m}[V_{r2}\, cos\theta_2 - V_{r1} cos\theta_1] \quad (4.47)$$

Let the force R_x has to be act opposite to F_c

$$R_x = -F_c = \dot{m}[V_{r1}\, cos\theta_1 - V_{r2} cos\theta_2] \quad (4.48)$$

Power developed by the vane is given by

$$P = uR_x = u * \dot{m}[V_{r1}\, cos\theta_1 - V_{r2} cos\theta_2] \quad (4.49)$$

From the outlet velocity triangle, it can be written

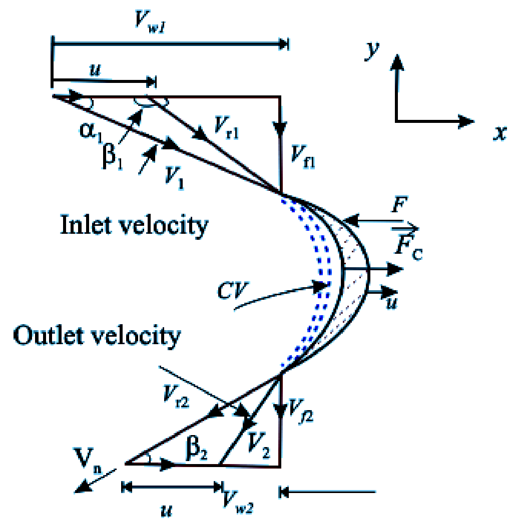


Figure 4.15: Flow of fluid along a moving curved plane

$$(V_{w2} + u)^2 = V_{r2}^2 - V_{f2}^2$$

Or $\quad V_{w2}^2 + u^2 + 2V_{w2}u = V_{r2}^2 - V_{f2}^2$

Or $\ V_2^2 - V_{f2}^2 + u^2 + 2V_{w2}u = V_{r2}^2 - V_{f2}^2$

Or $\quad V_{w2}u = \frac{1}{2}(V_{r2}^2 - V_2^2 - u^2) \quad (4.50)$

Similarly from the inlet velocity triangle. It is possible to write

$V_{w1}u = \frac{1}{2}(-V_{r1}^2 + V_1^2 + u^2)$ (4.51)

Addition of Eq's. (4.50 and 4.51) with no losses in relative velocity gives

$(V_{w1} + V_{w2})u = \frac{1}{2}(V_1^2 - V_2^2)$ (4.52)

Power of jet $= \dot{m}V_1^2/2$ (4.53)

The efficiency of the vane in developing power is given by

$\eta = \frac{out\ power}{input\ power} = \frac{uR_x}{\frac{\dot{m}}{2}V_1^2}$ (4.54)

Ex.9

A jet of water moving at 60 m/s is deflected by a vane moving at 25 m/s in a direction at 30° to the direction of the jet. The water leaves the blade normally to the motion of the vanes. Draw inlet and outlet triangle of velocities and find the vane angles for no shock at entering & exit. Take relative velocity at outlet equal to ($0.85V_{r1}$) and calculate the force on the vane of the jet diameter equal to (10) cm

Sol.

$u = 25\frac{m}{s}$; $V_1 = 60\frac{m}{s}$; $\propto_1 = 30°$

From triangle (ADC) as in below figure

$V_{w1} = 60\cos 30$

$V_{w1} = 60 * 0.866 = 51.96\frac{m}{s}$

$V_{f1} = V_1 \sin 30$

$V_{f1} = 60 * 0.5 = 30\ m/s$

$tan\theta_1 = \frac{CD}{AD - AB} = \frac{V_{f1}}{V_{w1} - U}$

$tan\theta_1 = \frac{30}{51.96 - 25} = 1.1127$

$\theta_1 = 48°4'$

$V_{r1} = \frac{V_{f1}}{sin\theta_1} = \frac{30}{0.7437} = 40.34\frac{m}{s}$

From triangle EFG

$V_{r2} = 0.85\,V_{r1} = 0.85 * 40.34 = 34.29\frac{m}{s}$

$cos\theta_2 = \frac{FG}{FE} = \frac{u}{V_{r2}} = \frac{25}{34.29} = 0.729$

$\theta_2 = 43°12'$

$\dot{m}_{r1} = \rho A V_{r1} = 1000 * \frac{\pi(0.1)^2}{4} * 40.34 = 316.7\ kg/s$

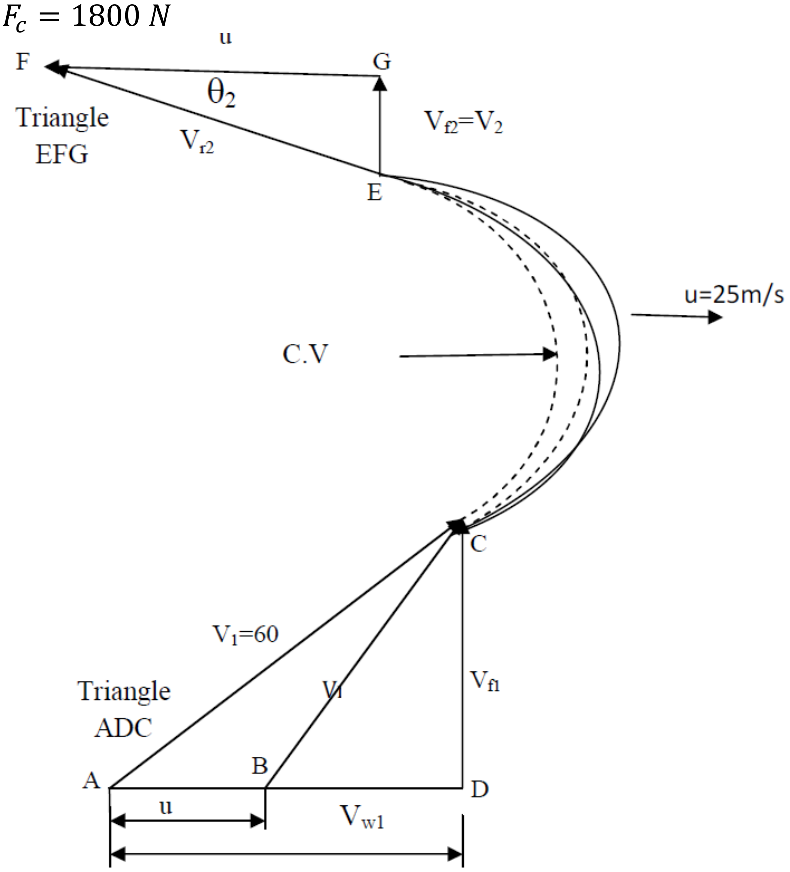
$\dot{m}_{r2} = \rho A V_{r2} = m_{r1} * 0.85 = 269.1\ kg/s$

$F_c = \dot{m}(V_{r2}cos\theta_2 - V_{r1}cos\theta_1)$

$F_c = 269.9 * 34.29\cos 43° - 316.7 * 40.34 * cos48°4'$

$F_c = -1800\ N$

$R = -F_c = 1800\ N$



III. ***Jet engine:***

A turbo jet engine as shown in Fig. 4.16 is consists essentially of

- a compressor
- a combustion chamber
- a gas turbine
- a nozzle

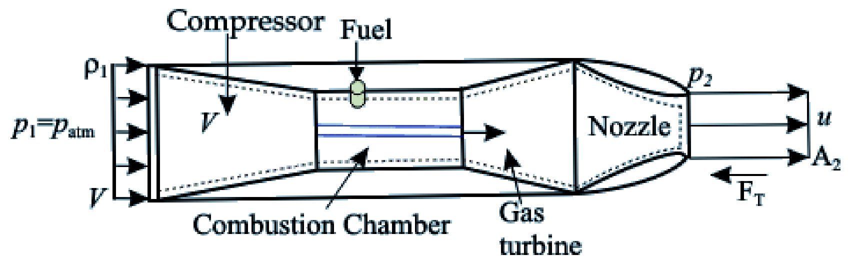


Figure 4.16: Turbo jet engine

Applying the momentum theorem for the C.V above as

$$(\dot{m}_a+\dot{m}_f)V_r-\dot{m}_aV=F_x-(p_2-p_{atm})A_2$$

Or $\quad F_x=(p_2-p_{atm})A_2+\dot{m}_a[(1+r)V_r-V] \qquad (4.55)$

$$r=\frac{mf}{ma}$$

Where F_x is the force acting on the C.V along the direction of the coordinate axis .

V =is the velocity of the aircraft

V_r = V_j-V is the relative velocity of the exit jet with respect to the aircraft

Vj= exit jet velocity of gas at nozzle as absolute

$\dot{m}_a$&$\dot{m}_f$ Are the mass flow rate of air and mass burning rate of fuel, usually $\dot{m}_f$ is very less compared to $\dot{m}_a$. $\dot{m}_f/\dot{m}_a$ usually varies from 0.01 to 0.02 in practice. The propulsive thrust on the aircraft can be written as

$$F_T=-F_x=-[(p_2-p_{atm})A_2+\dot{m}_a(V_r-V)]$$

Since $\dot{m}_f$ << $\dot{m}_a$, The propulsive power is given by

$$P=[\,\dot{m}_a(V_r-V)+(p_2-p_{atm})A_2]V \qquad (4.56)$$

The mechanical efficiency as the useful work divided by the same of useful work and kinetic energy as follows

$$\eta_m=\frac{output\ power}{input\ power},$$

$$\eta_m=\frac{[(p_2-p_{atm})A_2+\dot{m}_a(V_r-V)]V}{[(p_2-p_{atm})A_2+\dot{m}_a(V_r-V)]V+\frac{\dot{m}_a(Vr-V)^2}{2}}$$

$if\ p_2\approx p_{atm.}$

$$\eta_m=\frac{1}{1+\frac{(V_r-V)}{2V}} \qquad (4.57)$$

Ex.10

An airplane consumes 1 kg fuel for each 20kg air and discharge hot gases from the tail pipe at u=1800 m/s determine the mechanical efficiency for the airplane speeds of 300 m/s & 150 m/s when $p2\approx p_{atm}$ & $at\ V=300\frac{m}{s}$

Sol.

At V=300m/s ; V_r=V_j-V=1800-300=1500 m/s ; from Eq. 4.57

$$\eta_m=\frac{1}{1+\frac{(V_r-V)}{2V}}=\frac{1}{1+\frac{1200}{600}}=0.333=33.3\%$$

$$at\ V=150\frac{m}{s};\ V_r=1650\ m/s$$

$$\eta_m=0.1666=16.66\%$$

Ex.11

A jet engine under static test conditions in laboratory. Consumes 200 N/s air and 2 N/s fuel. What is the thrust produced from engine if the gas exit velocity is 450 m/s and the pressure at exit equal to atmosphere pressure.

Sol.

$F_t = -\dot{m}_a[(1+r)V_r - V] \quad r = \frac{2}{200} = 0.01;\ V = 0; V_r = V_j = 450\ m/s$

$F_t = -\dot{m}_a(1+r)V_r = -\frac{200}{9.81} * 1.01 * 450$

$F_t = -\ 9266\ N$

4.6.3 Angular Momentum (Moment of Momentum).

The moment of a force $\overrightarrow{F}\ about\ O$ is the vector or cross product

$$\vec{M} = \vec{r} \times \vec{F} \qquad (4.58)$$

Where $\vec{r}$ is the position vector from point 0 to any point on the line of action of $\vec{F}$. Vector product of two vector is a vector whose line of action is normal to the plane that contain the crossed vector ($\vec{r}$ & $\vec{F}$) from Fig.4.17 the magnitude of the moment of a force as

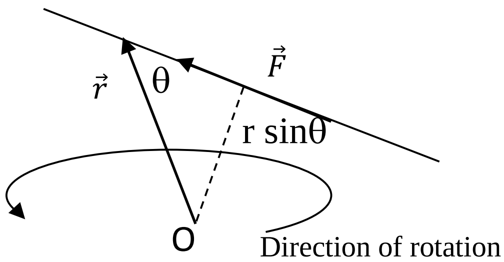


Figure 4.17: Moment of line force.

$$M = F\ r\ sin\theta \qquad (4.59)$$

Where θ is the angle between the lines of action of the vector $\vec{r}\ and\ \vec{F}$. Replacing the vector $\vec{F}$ in Eq. 4.58 by the moment vector $m\vec{V}$ gives the moment of momentum, and is called the angular momentum about O as

$$\vec{H} = \vec{r} \times m\vec{V} \qquad (4.60)$$

The angular momentum of differential mass $dm = \rho\ d\forall$ is

$d\vec{H} = (\vec{r} \times \vec{V})\rho\ d\forall$

$momentum\ of\ sys.\ \vec{H}_{sys} = \int_{sys}(\vec{r} \times \vec{V})\rho\ d\forall$

$\therefore \vec{r} \times \vec{V} \qquad is\ the\ angular\ momentum\ per\ unit\ mass$

The general C.V formulatims of the angular momentums is obtained from Eq. 4.19 by setting $N = \vec{H};\ \eta = \vec{r} \times \vec{V}$ in the general Reynolds Transport Theorem. Rate of change of moment of momentum as

$$\frac{dH_{sys}}{dt} = \frac{d}{dt}\int_{sys}(\vec{r} \times \vec{V})\rho d\forall \qquad (4.61)$$

$$\frac{dH_{sys}}{dt} = \frac{d}{dt}\int_{CV}(\vec{r} \times \vec{V})\rho\ dV + \int_{CS}(\vec{r} \times \vec{V})\rho(\vec{V}_r . \vec{n})dA$$

In general $\sum\vec{M} = \frac{d}{dt}\int_{CV}(\vec{r} \times \vec{V})\rho\ d\forall + \int_{CS}(\vec{r} \times \vec{V})\rho(\vec{V}_r . \vec{n})dA \qquad (4.62)$

$V_r = \vec{V} - \vec{V}_{cs}$; $\rho(\vec{V}_r.\vec{n})dA$ is the mass flow rate through dA into or out the C.V.
For fixed C.V $V_r = \vec{V}$

$$\sum \vec{M} = \frac{d}{dt}\int_{CV}(\vec{r} \times \vec{V})\rho\, dV + \int_{CS}(\vec{r} \times \vec{V})\rho(\vec{V}\ .\vec{n})dA$$

For steady flow

$$\sum \vec{M} = \int_{CS}(\vec{r} \times \vec{V})\rho(\vec{V}\ .\vec{n})dA \qquad (4.63)$$

The angular momentum flow rate can be expressed as the difference between the angular momentum of outgoing and incoming streams. If the flow is steady as well as uniform the angular momentum is

$$\sum \vec{M} = \sum_{out}(\vec{r} \times \vec{V})\dot{m} - \sum_{in}(\vec{r} \times \vec{V})\dot{m} \qquad (4.64)$$

In many problem, all the significant force and momentum flows are in the same plane, and then giving rise to moments in the same plane, Eq. 4.64 can be expressed in scalar from as

$$\sum M = \sum_{out} r\,\dot{m}V - \sum_{in} r\dot{m}V \qquad (4.65)$$

Where *r* represents the normal distance between the point about which moments are taken and the line of action of the force as velocity,

<u>Ex.12</u>

A small lawn sprinkler operates as indicated in figure. The inlet mass flow rate is 9.98 kg/min with inlet pressure of 30 kPa. The two exit jets direct flow at an angle of 40° above the horizontal. Determine the following

a) Jet velocity relative to the nozzle.
b) Torque required to hold the arm stationary.
c) Friction torque if the arm is rotating at 30 r.p.m.
d) Maximum rotational speed if we neglect friction

<u>Sol.</u>

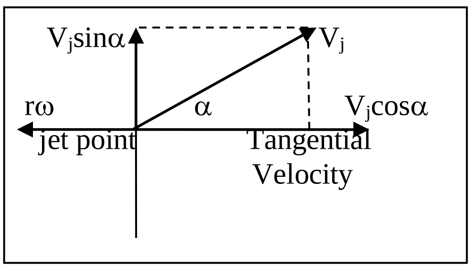
a) r_2=160 mm, d_j=5 mm

For each of the two jets :

$$Q_j = 0.5\frac{m_T}{\rho} = \frac{0.5*9.98}{1000} = 5*10^{-3}\ m^3/\min$$

$$A_j = \frac{\pi d_j^2}{4} = \frac{\pi(0.005)^2}{4} = 1.963*10^{-5}\ m^2$$

$$V_j = \frac{Q_j}{A_j} = \frac{5*10^{-3}}{60(1.963*10^{-5})} = 4.244 m/s$$



b) Torque required to hold the arm stationary taking the moment about the center of rotation

Inlet moment =0 due to r=0 the basic equation

$$\sum M = T_o = \sum_{out} \dot{m}_e(\vec{r} \times \vec{V}r) - \sum_{in} \dot{m}_i(\vec{r} \times \vec{V}r)$$

$\therefore T_o = 2\dot{m}_e\, r(V_j \cos \propto\ - r\omega)$

For stationary arm rω=0

$T_o = 2\, \rho Q_j\ r\ V_j \cos \propto$

$\therefore T_o = 2 * 1000 * \left(\frac{0.005}{60}\right) * 0.16 * 4.244 * \cos 40$

$T_o = 0.0866\, N.m$ counter clockwise (A resisting torque which must be applied in the counterclockwise direction to keep the arm from rotating in the clockwise direction).

c) At ω=30 rpm, calculate the friction torque T_f

$\omega = \frac{2\pi}{60} * 30 = \pi \frac{rad}{s}$

$T_o = 2\, \rho Q_j\ r\ (V_j \cos \propto - r\omega)$

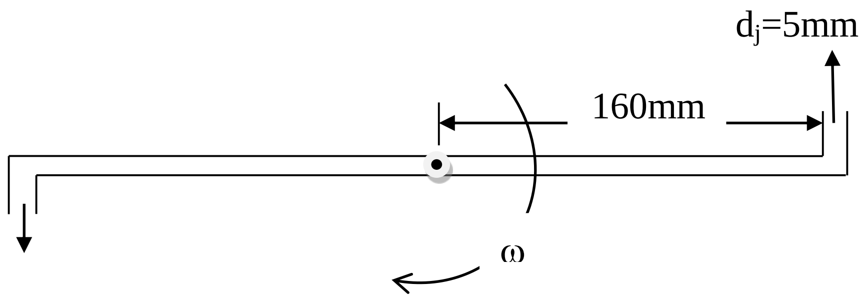
$T_o = 2 * 1000 * \left(\frac{0.005}{60}\right) * 0.16(4.244 \cos 40 - \pi * 0.16)$

$T_o = 0.07329\, N.m$

d) The maximum rotational speed occurs when the opposing torque is zero and all the moment of momentum goes to the angular rotation.

$V_j \cos \propto - r\omega = 0$

$\omega = \frac{V_j \cos\propto}{r} = \frac{4.244 * \cos 40}{0.16} = 20.319 \frac{rad}{sec} = 194\, rpm$



4.7 Radial-Flow Devices.

The fluid will be affected by centrifugal action of moving blades from the inner radius to the outer radius. Due to the suction created by the impeller motion, the fluid enters the eye of the impeller axially. The momentum transfer to the fluid by the impeller blades will increase the total head of the fluid and causing the fluid to flow out.

To analyze the centrifugal pump, we choose the annular region that encloses the impeller section as the *C.V*

ω: is the angular velocity of shaft impeller blades will have a tangential velocity

$V_{1,t} = \omega r_1$ at the inlet
$V_{2,t} = \omega r_1$ at the outlet (4.66)

For steady incompressible flow, the conservation of mass equation can be written as

$$\dot{Q}_1 = \dot{Q}_2 = \dot{Q} ---\rightarrow (2\pi r_1 b_1)V_{1,n} = (2\pi\, r_2 b_2)V_{2,n} \qquad (4.67)$$

Where b_1&b_2 are the flow widths at $r=r_1$ inlet & $r=r_2$ at outlet

The average normal components $V_{1,n}$ & $V_{2,n}$ of absolute velocity can be expressed in terms of the volumetric flow rate Q as

$$V_{1,n} = \frac{Q}{2\pi r_1 b_1} \quad \& \quad V_{2,n} = \frac{Q}{2\pi r_2\, b_2} \qquad (4.68)$$

Since $V_{1,n}$ & $V_{2,n}$ pass through the shaft center , thus they do not contribute to torque about the origin. Only the tangential velocity components contribute to torque and the application of the angular momentum as

$$\sum M = \sum_{out} r\,\dot{m}V - \sum_{in} r\dot{m}V \qquad (4.69)$$

$$\sum T_{shaft} = \dot{m}\left(r_2 V_{2,t} - r_1 V_{1,t}\right) \qquad (4.70)$$

Is known as Euler's turbine formula from Fig. 4.18

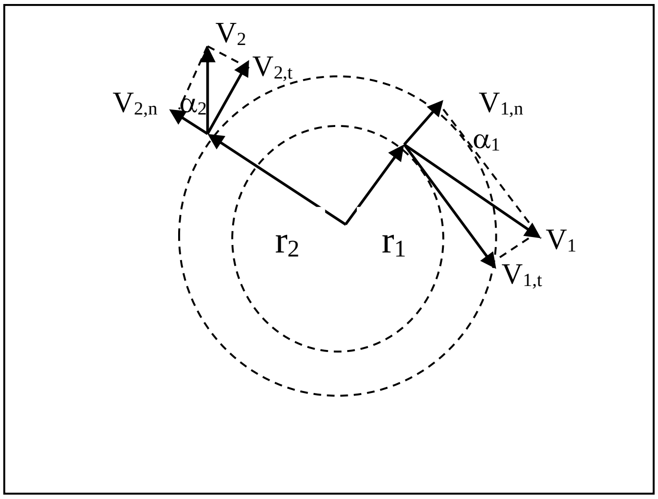


Figure 4.18: Velocities components in radial flow

$\propto_2$ & $\propto_1$ Angles between the direction of absolute flow velocities & the radial direction. Substituting Eq. 4.66 in Eq. 4.70 gives an ideal case when the tangential velocity begin equal to the blade angular velocity both at inlet & outlet

$$T_{sh,ideal} = \dot{m}\,\omega(r_2^2 - r_1^2)$$

Shaft power $P_{sh} = \omega\, T_{sh} = \frac{2\pi n}{60}\, T_{sh}$ (4.71)

Ex.13

Centrifugal blower has the following specifications.

$r_1 = 20cm\,, b_1 = 8.2cm \quad at\ inlet$

$r_2 = 45cm\ \,, b_2 = 5.6cm\ at\ outlet$

$Q = 0.7\frac{m^3}{s}$, $n = 700\ r.p.m$

$\propto_1 = 0°\ at\ inlet \quad \propto_2 = 50°\ from\ radial\ direction$

Determine the minimum power consumption of the blower $\rho_{air} = 1.25 kg/m^3$

Sol.

$\sum M = \sum_{out} r\,\dot{m}\vec{V} - \sum_{in} r\,\dot{m}\vec{V}$

$Q_1 = Q_2 = Q = 0.7\ m^3/s\ , \quad \dot{m} = \rho * Q = 1.25 * 0.7 = 0.875\ kg/s$

$V_2 = \frac{Q}{A2} = \frac{0.7}{(2\pi r2 * b2)} = \frac{0.7}{(2\pi * 0.45 * 0.056)} = 4.42\frac{m}{s}$

$T_{sh} = \dot{m}(r_2\ V_2 sin \propto 2 - r_1 V_1 sin \propto 1)$

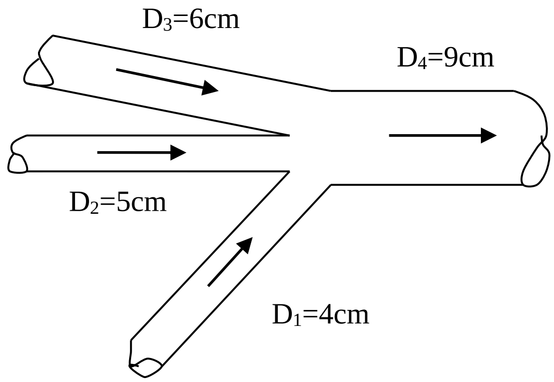
$= 0.875(0.45 * 4.42 * sin50 - 0) = 1.33 \quad N.m$

$P = \omega.T_{sh} = \frac{2\pi n}{60}\ T_{sh} = \frac{2\pi * 700}{60} * 1.33 = 97.75\ W$

Problems.

P4.1 A pipe of diameters (***12cm***) and (***8cm***) through sections (1) and (2) respectively. If the velocity at section (1) is (***1.5 m/s***) , what is the velocity at section(2)?

P4.2 the velocity of flow ($V_2 = 5\frac{m}{s}$) and the exit flow rate ($Q_4 = 120\frac{m^3}{h_r}$) as in below figure. Find a) V_1 , b) V_3 and c) V_4 if it's known that increasing in Q_3 by (***20%***) would increase Q_4 by (***20%***).



P4.3 The velocity distribution for a (***3-D***), incompressible, steady state flow is given by:

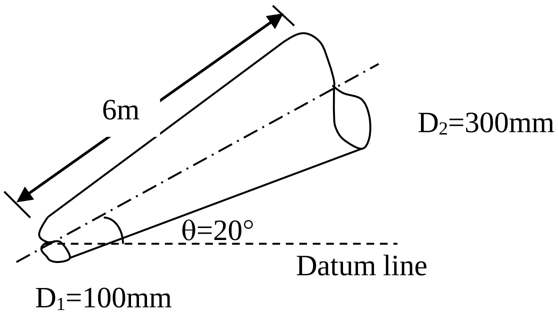
$u = 2x^2 - xy + z^2$

$v = x^2 - 4xy + y^2$

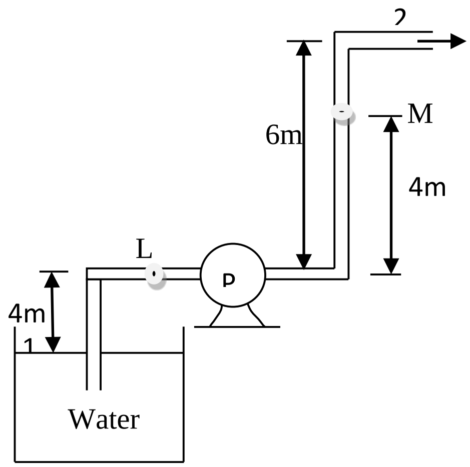
$w = -2xy - yz + y$

Show that it satisfies the continuity equation in (***3-D***).

P4.4 A ***6m*** long pipe is inclined at an angle of ***20°*** as shown in figure. If the velocity at smaller section is ***1.8 m/s***, find the difference of pressure between the two sections.



P4.5 As shown in the figure, the pump is lift water at ***60 lit/s***, through ***0.1m*** diameter. Determine the required power by assuming the overall efficiency of ***70%***. Also, find pressure intensities at points (***L***) and (***M***).



P4.6 Determine whether the continuity equation is satisfied by the following velocity components for an incompressible fluid.

$u_2 = x^2y$ $\quad v_2 = 2xy - xy^2$ $\quad w_2 = x^2 - z^2$

P4.7 A pipe (1) has diameter is ***450mm*** branches into two pipes (***2&3***) of diameters ***300mm*** and ***200mm*** respectively. If the a average velocity in pipe (1) ***3m/s*** find

i) discharge through pipe (1)

ii) Velocity in pipe (3) if the average velocity in pipe (2) is ***2.5 m/s***.

P4.8 The tangential component of velocity in a two dimension flow incompressible fluid is $V_\theta = -\frac{Csin\theta}{r^2}$ where C is constant.

i) Using continuity equation. Determine the expression for radial velocity V_r.

ii) Find the magnitude of resultant velocity.

P4.9 A cube tank has side ***2m***. The tank filled to level (***h***) above orifice that placed above the its base, with ***0.06m*** diameter and ***0.7*** coefficient discharge.

i) If water enters the tank at a constant rate $\left(0.015\frac{m^3}{s}\right)$ find the depth of water above the orifice when the level in the tank becomes constant.

ii) Find the time when the level of water fall from ***4m*** to ***0.5m*** above the orifice when the flow in is turn off.

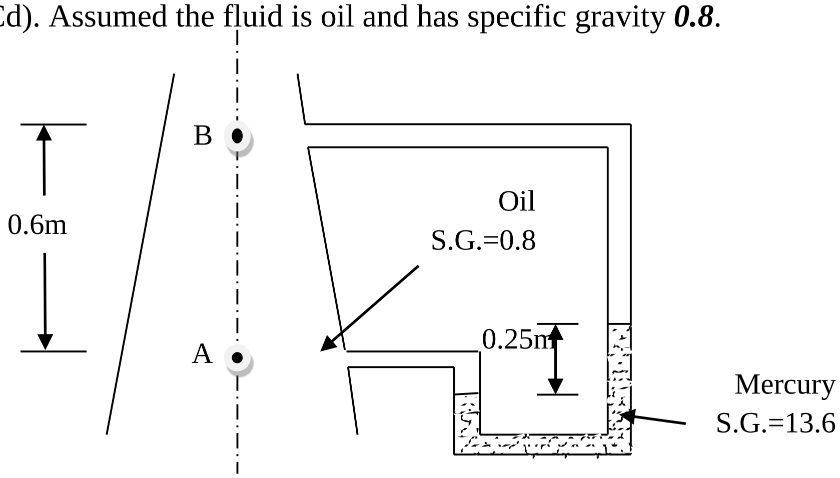
P4.10 A horizontal cylinder ***2m*** diameter and ***10m*** long is half full of water. Find the time of emptying the cylinder through a short opining pipe of diameter ***0.08m*** attached to the bottom of cylinder. Take the coefficient of discharge to be (***cd=0.8***)

P4.11 Water flows in pipe (1) with diameter ***0.75m*** and branches into three pipes (2, 3, 4) with diameters (***0.3, 0.3, 0.4***) m respectively

i) If the Velocity of a pipe 1 is ***1.2 m/s*** find the flow rate at pipe

ii) If the velocity at (2&3) is 2 ***m/s*** find the velocity at pipe 4.

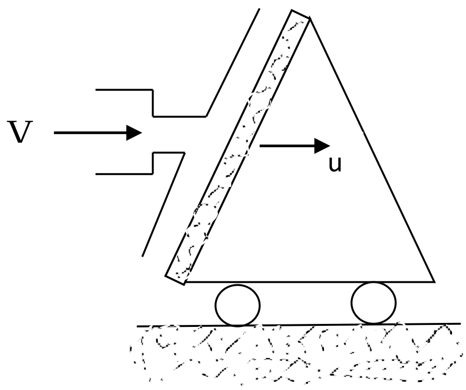
P4.12 The vertical pipe as shown in below figure. The pipe Convergent and connected to U tube manometer. The diameters at ***A*** section is ***0.3m*** and at ***B*** section is ***0.15m***. Find the flow rate in the pipe if the actual flow rate in this pipe is $0.15\frac{m^3}{s}$. Also, find the coefficient of discharge (Cd). Assumed the fluid is oil and has specific gravity ***0.8***.



P4.13 water enters ***Y*** reducing horizontal pipe and comes out vertical in the downward direction. If the inlet velocity is ***5 m/s*** and pressure is ***80kPa*** (gauge) and diameter at entrance and exit section are ***30cm*** and ***20cm*** respectively. Calculate the component of reaction acting on the pipe.

P4.14 360 liters per second of water is flowing in a pipe. The pipe is bent by 120°. The pipe measures diameters (***360 mm*** and ***240 mm***) and volume at the bend is ***0.14m³***. The pressure at the entrance is 73kN/m^2 and exit is 2.4 m above the entrance section.

P4.15 A flat plate is moving with Velocity ***u*** in a same direction at the jet as shown in below figure. Drive the expression for power developed due to motion at the plate.

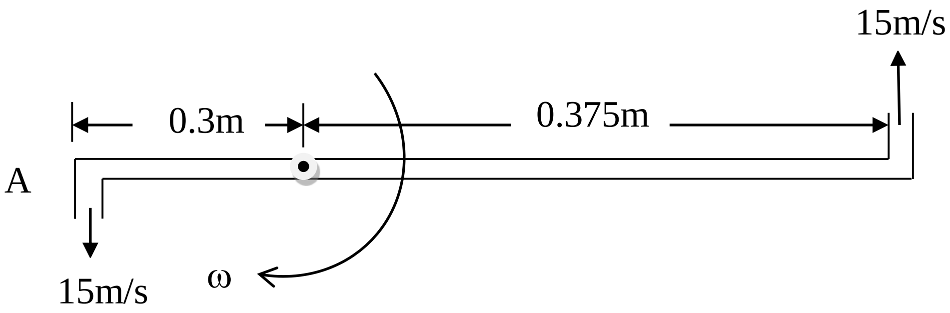


P4.16 The water is flowing in a pipe having a diameter of ***300 mm*** with flow rate ***250 l/s***. If the pipe is bent by ***133°***, find the magnitude and direction of the resultant force on the bend. The pressure of the water flowing is ***400 kN/m²***.

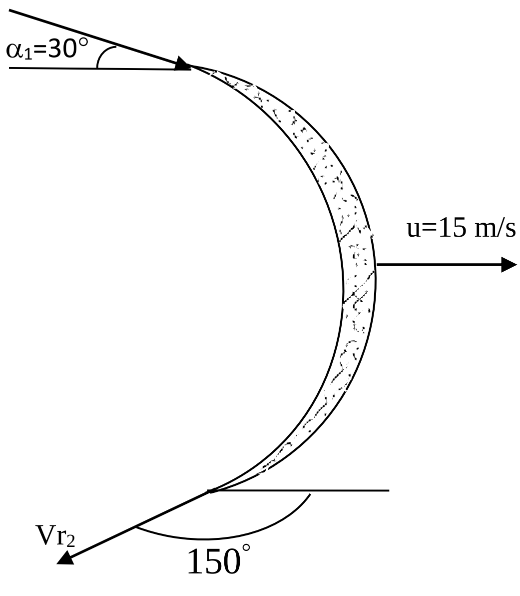
P4.17 A sprinkler in below figure has ***12mm*** diameter nozzles at the end of rotation arm and discharge water with a velocity of ***15 m/s***. Determine

i) Torque required for holding the rotating arm stationary.

ii) Maximum rotational speed if we neglect friction.



P4.18 **A** jet of water exits from nozzle with $V_1 = 25\frac{m}{s}$,$\alpha_1 = 30°$ with horizontal as in below figure and $\dot{m} = 15\frac{kg}{s}$. If the angle at exit of water from the vane is 150° with horizontal and the water losses is 15% from its velocity by friction with surface of vane compute the power developed by the vane.



CHAPTER 5

Applications of energy Equation

5.*1 Measurement of Flow Rate Through Pipe*.

The determination of the flow rate from the measurement of pressure drop depends on the straight forward application of Bernoulli's equation. Four different flow meters operate on this principle.

- Venturimeter
- Orificemeter
- Flow nozzle
- Pitot tube

5.1.1 Venturimeter.

Working:

1- The gradual diverging passage in the direction of flow avoiding the losses of energy due to separation

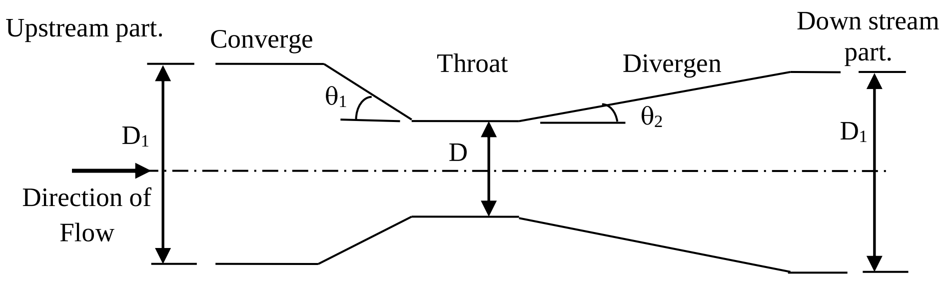


Figure 5.1: Venturimeter.

2- $\frac{dV}{dx} > 0$, $\frac{dp}{dx} < 0$ → To satisfy continuity equation in convergent part according to Bernoulli's Equation

3- $V_{max.}$, p_{min} → At throat area, demonstrated by Battisla Venturi 1797.

4- $dV/dx<0$, $dp/dx>0$ → In the divergent part.

For measuring the flow rate through pipe, let us consider a steady, ideal and one dimensional, where the flow of fluid, the velocity and pressure at any section will be uniform. Let V_1&p_1 are the velocity and pressure at inlet section (1), while those at throat V_2 & p_2 at section (2) as shown in Fig. 5.2. Applying Bernoulli's equation between sec.1&2, we get

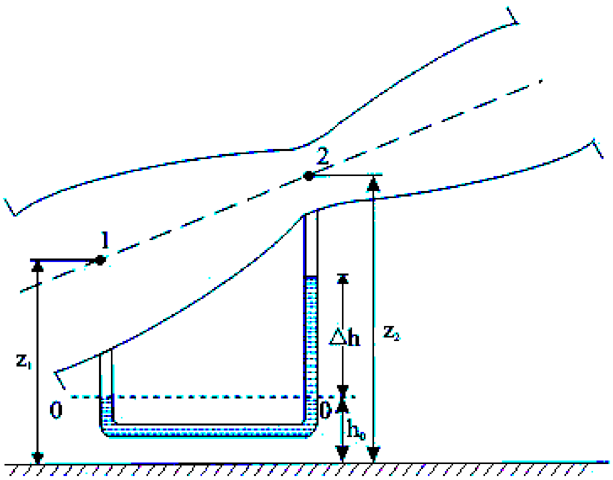


Figure 5.2: Measurement of Flow by a Venturimeter.

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p2}{\rho g}+\frac{V_2^2}{2g}+z_2 \qquad (5.1)$$

$$\frac{V_2^2-V_1^2}{2g}=\frac{p_1-p_2}{\rho g}+z_1-z_2 \qquad (5.2)$$

Where ρ is the density of fluid through the Venturimeter. From continuity

$$A_1V_1=V_2A_2 ----\rightarrow V_1=\frac{V_2A_2}{A_1} \qquad (5.3)$$

Subtituting Eq. 5.3 in Eq. 5.2

$$\frac{V_2^2}{2g}\left(1-\frac{A_2^2}{A_1^2}\right)=\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)$$

$$V_2=\frac{1}{\sqrt{1-\frac{A_2^2}{A_1^2}}}\sqrt{2g(h_1^*-h_2^*)} \qquad (5.4)$$

Where h_1^*&h_2^* are the piezometric pressure at Sec.1 & Sec.2 and are defined as

$$h_1^*=\frac{p_1}{\rho g}+z_1 \qquad (5.5.a)$$

$$h_2^*=\frac{p_2}{\rho g}+z_2 \qquad (5.5.b)$$

Hence, the volume flow rate through the pipe is given by

$$Q = A_2\ V_2 = \frac{A_2}{\sqrt{1-\frac{A_2^2}{A_1^2}}}\ \sqrt{2g(h_1^* - h_2^*)} \tag{5.6}$$

The pressure difference between Sec.1&2 is measured by a manometer as shown in Fig 5.2, we can write

$$p_1 + \rho g(z_1 - h_0) = p_2 + \rho g(z_2 - h_0 - \Delta h) + \Delta h\,\rho_m g$$

$$or, (p_1 + \rho g z_1) - (p_2 + \rho g z_2) = (\rho_m - \rho) g \Delta h$$

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h$$

$$or\ \ h_1^* - h_2^* = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h \tag{5.7}$$

Where ρ_m is the density of the manometric liquid. Eq. 5.7 shows that a manometer always registers a direct reading of the difference in piezometric pressures. Now, substitution of $(h_1^* - h_2^*)$ from Eq. 5.7 in Eq. 5.6 will gives the flow rate through pipe.

$$Q = \frac{A_1 A_2}{\sqrt{A_1 - A_2}}\ \sqrt{2g(h_1^* - h_2^*)} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\ \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \tag{5.8}$$

If C the constant of Venturimeter which is equal to $\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\ \sqrt{2g}$ and the pipe along with the Venturimeter is horizontal, then $z_1 = z_2$, and hence $h_1^* - h_2^*$ becomes$(h_1 - h_2)$, where h_1 and h_2 are the static pressure heads can be written as $(h_1 = \frac{p_1}{pg}\ , h_2 = \frac{p_2}{pg})$ then, the manometric Eq. 5.7 becomes

$$h_1 - h_2 = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h$$

Eq. 5.8 gives the flow rate in pipe with the terms of manometer deflection Δh is remain the same irrespective of whether the pipe-line along with the Venturimeter connection is horizontal or not. Eq. 5.8 always overestimates the actual flow rate due to the ideal flow assumption and read fluid measurement *(Δh)*. Multiplying Eq. 5.8 by the factor C_d, called the coefficient of discharge as follows.

$$Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\ \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \tag{5.9}$$

C_d <1.0 and is defined as

$$C_d = \frac{Actual\ rate\ of\ discharge}{theoretical\ rate\ of\ discharge\ by\ Eq5.8}$$

Value of C_d between (0.95 to 0.98); $Cd \approx 0.9858-0.196\ \beta^{4.5}$ where $\beta=(d2/d1)$

Ex.1

A Venturimeter is placed at 30° to the horizontal (sloping upwards in the direction of flow) to a pipe line carrying on oil of specific gravity 0.8. A differential with mercury as the manometer fluid is attached to the inlet and throat of the Venturimeter. The manometer shows a deflection of 100 mm. the pipe diameter is 200 mm, while the diameter of Venturi throat is 100 mm.

a) Find the volume flow rate of oil if the coefficient of discharge of the Venturimeter is 0.96.
b) What will be the reading of differential manometer if the Venturimeter is turned horizontal? The length of Venturimeter between the inlet and the throat is 320 mm.

Sol.

$A_1 = 0.0314\, m^2;\ A_2 = 0.00785\, m^2$

$$Q_{act} = Cd\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\Delta h} \qquad \rho = 800\frac{kg}{m^3},\ \rho_m = 13600\frac{kg}{m^3}$$

$\Delta h = 0.1m, Cd = 0.96$

a) $Q_{aut} = 0.04386\frac{m^3}{s}$

b) $V_1 = \frac{Q}{A_1} = \frac{0.04386}{0.0314} = 1.388\frac{m}{s}$

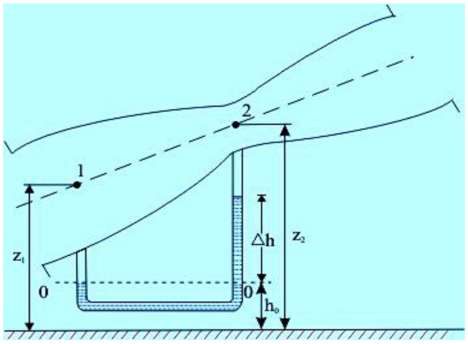
$V_2 = \frac{Q}{A_2} = \frac{0.04381}{0.00785} = 5.58\frac{m}{s}$

$IF\ z_1 = z_2$

$$\frac{p_1 - p_2}{Pg} = \frac{V_2^2 - V_1^2}{2g} = \frac{5.50^2 - 1.382^2}{2*9.81} = \frac{31.38 - 1.96}{2*9.81} = 1.488\, m$$

From Eq. 5.7 $\frac{p_1 - p_2}{\rho g} = h_1 - h_2 = (\frac{\rho_m}{\rho} - 1)\Delta h$ where $z_1 = z_2$

$$\frac{p_1 - p_2}{\rho g} = \left(\frac{\rho_m}{\rho} - 1\right)\Delta\overline{h} = 1.488m \qquad \Delta\overline{h} = \frac{1.488}{16} = 0.093m$$



5.1.2 *Orificemeter.*

A- First Method

Is a cheaper arrangement for the measurement of flow through a pipe, is essentially a thin circular plate with a sharp edged concentric circular hole in it as in Fig. 5.3.

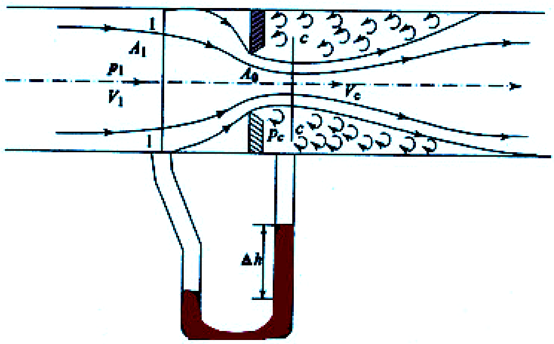


Figure 5.3: Flow through an Orificemeter.

Consider the fluid to be ideal, by applying Bernoulli's theorem between Sec.1-1 and Sec. c - c

$$\frac{p_1^*}{\rho g}+\frac{V_1^2}{2g}=\frac{p_C^*}{\rho g}+\frac{V_C^2}{2g} \qquad (5.10)$$

Where p_1^* & p_2^* are the piezometric pressure at Sec. 1-1 & c - c respectively. From continuity equation

$$V_1A_1 \approx V_cA_c \qquad (5.11)$$

Where A_c is the area of the *vena contracta* from Eq's. 5.10 & 5.11 we can written as,

$$V_c=\sqrt{2(p_1^*-p_c^*)/\rho\left(1-\frac{A_C^2}{A_1^2}\right)} \qquad (5.12)$$

The measured value of the piezometric pressure drop for a real fluid is always more due to friction than that assumed in case of an inviscid flow, a coefficient of velocity C_V (always less than 1) has to be introduce to determine the actual velocity V_c when the pressure drop $p_1^*-p_c^*$ in Eq. 5.12 is substituted by its measured value in terms of the monometer deflection Δh.

$$\Delta p=(\rho_{merc}-\rho_{water})g\Delta h=\rho g\left(\frac{\rho_m}{\rho}-1\right)\Delta h.$$

Hence,

$$V_c=C_V\sqrt{\frac{2g\left(\frac{\rho_m}{\rho}-1\right)\Delta h}{1-A_c^2/A_1^2}} \qquad (5.13)$$

Where

Δh is the difference in liquid level.

ρ_m is the density of the manometric liquid.

ρ is the density of the working fluid.

$\therefore$ Volumetric flow rate

$$Q=A_c\,V_c \qquad (5.14)$$

If a coefficient of contraction C_c is defined as $C_c=\frac{A_c}{A_2}$, $\quad A_c=C_c\,A_2$

A_2 Is the area of orifice due to unknown the position of $A_c$along the flow. Eq. 5.14 can be written with help of Eq. 5.13.

$$Q = C_c\, A_2\, C_V \sqrt{\frac{2g\left(\frac{\rho m}{\rho}-1\right)\Delta h}{1-\frac{C_C^2 A_2^2}{A_1^2}}}$$

$$Q = C_c A_2 C_V \sqrt{\frac{2g}{1-\frac{C_C^2 A_2^2}{A_1^2}}}\ \sqrt{\left(\frac{\rho m}{\rho}-1\right)\Delta h}$$

$$Q = C_d \sqrt{\left(\frac{\rho m}{\rho}-1\right)\Delta h} \qquad (5.15)$$

With, $C_d = C \quad A_2 \sqrt{\frac{2g}{1-\frac{C_C^2 A_2^2}{A_1^2}}}$ $, Where\ (C \quad = C_V C_c)$

Where ***C*** is depends upon the ratio of orifice to duct area, and Reynolds number of flow.

B- Orificemetes (Second Method)

A_1, V_1, p_1 at Sec.1 A_2, V_2, p_2 at Sec.2. Applying ***B.E.*** at Sec.1 & 2 we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1 = \frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right) = \frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

$$\Delta h^* = \frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

$$\frac{V_2^2}{2g} = \Delta h^* + \frac{V_1^2}{2g}$$

$$or\ V_2 = \sqrt{2g\left(\Delta h^* + \frac{V_1^2}{2g}\right)} = \sqrt{2g\,\Delta h^* + V_1^2} -----(a)$$

Section 2 is at *vena contracta* and A2 represents the area of vena contracta, ***Ao*** is the area of orifice,

$C_C = \frac{A_2}{A_0}$ Where C_C =Co-efficient of contraction

$\therefore\ A_2 = C_C A_0 ------(b)$ `

Using C.E. ,we get

$A_1 V_1 = A_2 V_2 ---\rightarrow OR\ V_1 = \frac{A_2 V_2}{A_1}$

$Or \quad V_1 = \frac{A_0 C_C V_2}{A_1} -------- (c)$

Substituting the value of V_1 Eq. (a), we get

$$V_2 = \sqrt{2g\,\Delta h^* + A_0^2 C_C^2 . \frac{V_2^2}{A_1^2}}$$

$$Or\ \ V_2^2 = 2g\Delta h^* + \left(\frac{A_0}{A_1}\right)^2 . C_C^2 . V_2^2$$

$$V_2^2\left[1-\left(\frac{A_0}{A_1}\right)^2 C_C^2\right]=2g\ \Delta h^*$$

$$V_2=\frac{\sqrt{2g\ \Delta h^*}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}}$$

$$\therefore The\ discharge\quad Q=A_2V_2=A_0.C_cV_2\ =A_0C_C\frac{\sqrt{2g\Delta h^*}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}}----(d)$$

The above expression is simplified by using

$$C_d=C_C\frac{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}}\qquad C_d=coefficient\ of\ discharge$$

$$C_C=C_d\frac{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_C^2}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}}\qquad C_d=C_C.C_V$$

Substituting the value of C_C in Eq. d, we get

$$Q=A_0.C_d\ \frac{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}}*\frac{\sqrt{2g\Delta h^*}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2 C_c^2}}\ ;\ Q=\frac{C_dA_0\sqrt{2g\Delta h}}{\sqrt{1-\left(\frac{A_0}{A_1}\right)^2}}=\frac{C_dA_0A_1\sqrt{2g\Delta h^*}}{\sqrt{A_1^2-A_0^2}}$$

$\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)=\Delta h^*=h_1^*-h_2^*=\left(\frac{\rho_m}{\rho}-1\right)\Delta h$ is the differential head.

$$Q=C_d\frac{A_0A_1}{\sqrt{A_1^2-A_0^2}}\ \sqrt{2g(\frac{\rho_m}{\rho}-1)\Delta h}\qquad (5.16)$$

Ex.2

The following date related to an orificemeter

Diameter of pipe =240 mm

Diameter of orifice =120mm

Reading of differential manometer =400 mm of mercury Co-efficient of discharge of the meter =0.65. Determine the rate of oil flow

Sol.

$d_1=240mm=0.24\ m$

$\therefore Area\ of\ pipe\ A_1=\frac{\pi}{4}*0.24^2=0.0452m^2$

$orifice\ diameter\ d_o=120\ mm=0.12m$

$A_0=\frac{\pi}{4}*0.12^2=0.0113m^2$

$C_d=0.65$

$S.G_{oil} = 0.88$

Reading differential *h*= 400mm = 0.4 m of mercury

$\therefore differential\ head = \Delta h^* = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$

$\therefore\ \Delta h^* = 0.4\left[\frac{13.6}{0.88} - 1\right] = 5.78\, m\ of\ oil$

$Q = C_d\ \frac{A_0 * A_1\sqrt{2g\,\Delta h^*}}{\sqrt{A_1^2 - A_0^2}}$

$Q = 0.65 * \frac{0.0113 * 0.0452\sqrt{2 * 9.81 * 5.78}}{\sqrt{0.0452^2 - 0.0113^2}}$

$Q = 0.08\frac{m^3}{s}$

5.1.3 Flow Nozzle.

- The flow nozzle as shown in Fig.15.4 is essentially a Venturimeter with the divergent part omitted. Therefore the basic equations for calculation of flow rate are the same as those for a Venturimeter.
- The dissipation of energy downstream of the throat due to flow separation is greater than that for a Venturimeter. But this disadvantage is often offset by the lower cost of the nozzle.
- The downstream connection of the manometer may not necessarily be at the throat of the nozzle or at a point sufficiently far from the nozzle.
- The deviations are taken care of in the values of C_d, The coefficient C_d depends on the shape of the nozzle, the ratio of pipe to nozzle diameter and the Reynolds number of flow.

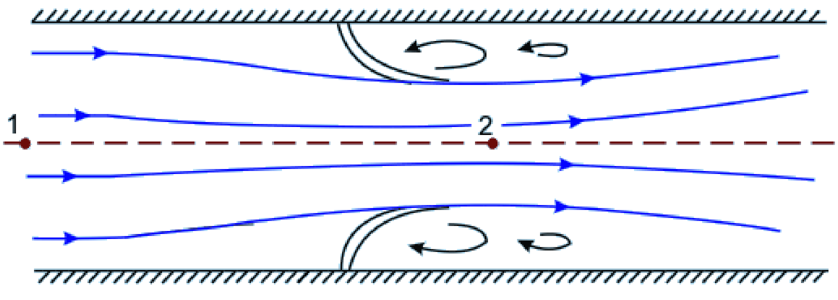


Figure 5.4: A flow nozzle.

- A comparative picture of the typical values of C_d, accuracy, and the cost of three flow meters (venturimeter, orificemeter and flow nozzle) is given below:

Type of Flowmeter	Accuracy	Cost	Loss of Total Head	Typical Values of C_d
Venturimeter	High	High	Low	0.95 to 0.98
Orificemeter	Low	Low	High	0.60 to 0.65
Flow Nozzle	Intermediate between a venturimeter and an orificemeter			0.70 to 0.80

5.2 *Orifice in a Reservoir.*

(*h*) is the head is measured from the center of the orifice to the free surface as in Fig. 5.5. Bernoulli's Eq. applied from a point (1) on the free surface to the center of the *vena contracta* point (c). Neglecting losses, is written

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_c}{\rho g}+\frac{V_c^2}{2g}+z_c$$

$h=\frac{V_c^2}{2g}\longrightarrow V_C=\sqrt{2gh}$ also $V_{ci}=\sqrt{2gh}$ where V_{ci} is the theoretical velocity.

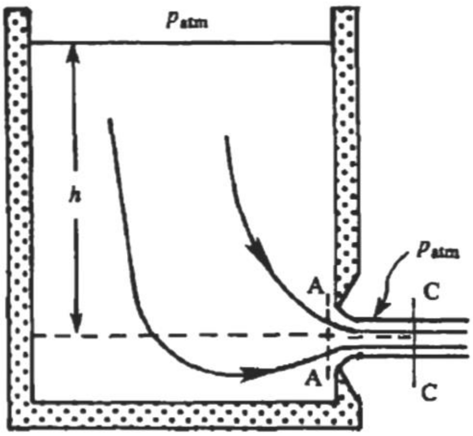


Figure 5.5: Flow through a sharp edge orifice.

To calculate the discharge from orifice in reservoir, we must find the actual velocity (V_{ca}). C_V is the coefficient of velocity

$$C_V=\frac{V_{ca}}{V_{ci}}\longrightarrow V_{ca}=C_V \quad V_{ci}$$

$$V_{ca}=C_V\sqrt{2gh}$$

To calculate the actual flow rate $A_c=C_cA_2$, where A_2 is the orifice area

$$\therefore Q_{act}=A_cV_{ca}=C_c\,C_VA_2\sqrt{2gh}$$

C_d is the coefficient of discharge

$C_d = C_c C_V \qquad or\ C_d = \frac{Q_{act}}{Q_i}$

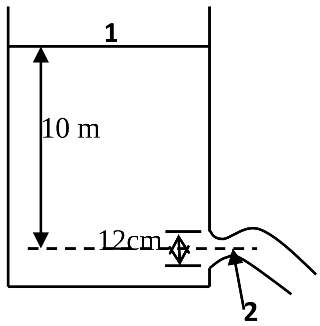
$Q_{act} = C_d A_2 \sqrt{2gh}$

$Q_{act} = C_d\, Q_i$

Ex.3

As in figure the orifice diameter is ***(12 cm)*** in reservoir and the level of water above the orifice is ***(10 m)***. Calculate the actual flow rate when the coefficient of discharge is ***(0.65)***.

Sol.



Applying B.E. between point ***1 & 2***.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_{ci}^2}{2g} + z_2$$

$$10 = \frac{V_{Ci}^2}{2*9.8}$$

$$V_{ci} = \sqrt{10*2*9.81} = 14.0\,\frac{m}{s}$$

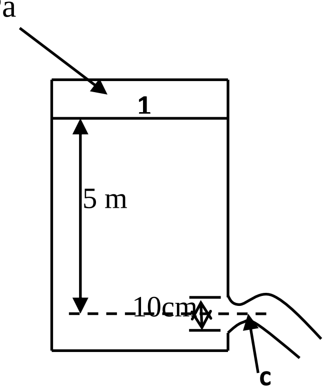
$$Q_i = A_2 * V_{ci} = \frac{\pi}{4}(0.12)^2 * 14 = 0.15833\,\frac{m^3}{s}$$

$$Q_{act} = C_d Q_i = 0.65 * 0.15833 = 0.1029\,\frac{m^3}{s}$$

Ex.4

Calculate the actual flow rate from the orifice diameter ***(10 cm)*** in reservoir forming a *vina contracta* diameter ***(8.5 cm)*** and the (C_V & C_C) as in figure. Take the discharge coefficient $C_d = (0.58) \& S.G. = 0.9$

Sol.



Apply B.E. between (1&c)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c$$

$$\frac{35*10^3}{0.9*9810} + 5 = \frac{V_{ci}^2}{2*9.81} ----\rightarrow V_{ci} = 13.26\,\frac{m}{s}$$

$$V_{2i} = V_{ci} = 13.26\,\frac{m}{s}$$

$$Q_i = A_2 * V_{2i} = \frac{\pi}{4} * (0.1)^2 * 13.26 = 0.1041\,\frac{m^3}{s}$$

$$Q_{act} = C_d * Q_i = 0.58 * 0.1041 = 0.0604\,\frac{m^3}{s}$$

$$C_c = \frac{A_c}{A_2} = \frac{\frac{\pi}{4}(0.085)^2}{\frac{\pi}{4}(0.1)^2} = 0.7225$$

$$C_d = C_V C_C ----\rightarrow C_V = \frac{C_d}{C_C} = \frac{0.58}{0.7225}$$

$$C_V = 0.8$$

5.3 Pitot Tube for Flow Measurements.

5.3.1 Hydrostatic, Hydrodynamic, Static and Total Pressure.

The points ***A&B*** are at a height $z_A \& z_B$ respectively from the datum & consider a fluid flow through pipe of varying cross section area as in Fig. 5.6.

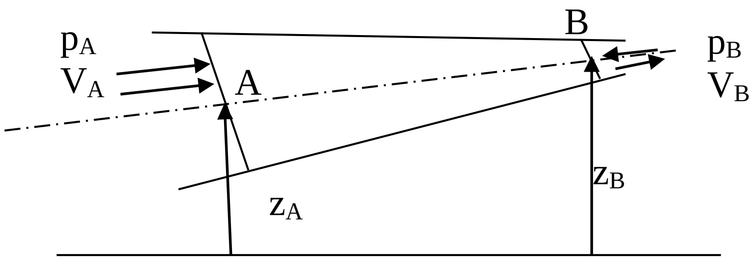


Figure 5.6: Static and Total Pressure.

- If the fluid is to be stationary , then $(\frac{\partial p}{\partial z})_{hs} = -\rho g$

(hs) Represent the hydrostatic case.

So, $p_{Ahs} - p_{Bhs} = \rho g(z_B - z_A)$

From above equation, the hydrostatic pressure at a point in a fluid is the pressure acting at the point when the fluid is at rest or pressure at the point due to weight of the fluid above it.

- Now, if the fluid to be moving, the pressure at a point can be written as a sum of two components, hydrodynamic & hydrostatic

$$p_A = p_{Ahs} + p_{Ahd} \qquad (5.17)$$

- Using Eq. 5.17 in Bernoulli's equation between ***A&B***

$$\frac{p_{Ahd} - p_{Bhd}}{\rho g} + \left[\frac{p_{Ahs} - p_{Bhs}}{\rho g} + (z_A - z_B)\right] = \frac{V_B^2 - V_A^2}{2g} \qquad (5.18)$$

From Eq. 5.18, the terms within the square bracket cancel each other, hence

$$\frac{p_{Ahd} - p_{Bhd}}{\rho g} = \frac{V_B^2 - V_A^2}{2g} \qquad (5.19)$$

$$p_{Ahd} + \frac{\rho V_A^2}{2} = p_{Bhd} + \frac{\rho V_B^2}{2} = C = p_O \qquad (5.20)$$

Eq's (5.19 &5.20) convey the flowing

The pressure at a location has → Hydrostatic component
→ Hydrodynamic component

The difference in kinetic energy due to hydrodynamic components only.

Note.

1- The hydrodynamic component is often called static pressure.
2- The velocity term is the dynamic pressure.
The sum of two components is (p_O) is known as total pressure.

$$p_O = p + \frac{\rho V^2}{2} \qquad (5.21)$$

Is known as stagnation pressure

$$V = \sqrt{2\frac{p_O - p}{\rho}} \qquad (5.22)$$

5.3.2 Pitot Tube for Measurement.

Firstly at 1732 by Henri Pitot, was used a right angled glass tube, one end of the tube face the flow while the other end is open to atmosphere. The difference in level between the liquid in the glass tube and the free surface becomes the measure of dynamic pressures neglecting friction as in Fig. 5.7.a.

$$p_O - p = \frac{\rho V^2}{2} = \rho g\, h$$

$$V = \sqrt{2gh}$$

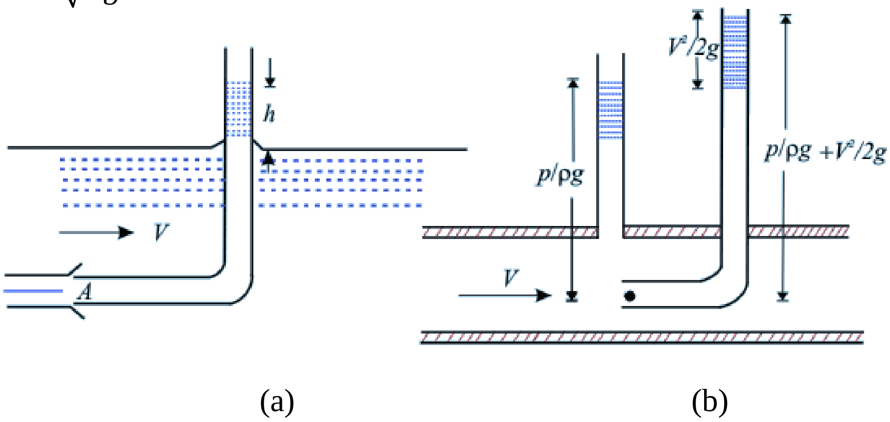


Figure 5.7: Simple Pitot tube (a) Tube for measuring the stagnation pressure.
(b) Static and stagnation tubes together.

For a free stream a single tube is sufficient to determine the velocity.In closed duct the Pitot tube measures only the stagnation pressure and so the static pressure must be measured separately as shown in Fig. 5.7-b. Applying B.E. between stagnations s & p in horizontal pipe

$$\frac{p_O}{\rho g} + \frac{V^2}{2g} = \frac{p_s}{\rho g} \quad or \qquad h_o + \frac{V^2}{\rho g} = h_s$$

$$V = \sqrt{2g(h_s - h_o)} = \sqrt{2g\Delta h}$$

Where:

p_o= Pressure at point p. i.e. static pressure.

V= Velocity at point (p) i.e free flow velocity

p_s= Stagnation pressure at point s

Δh= Dynamic pressure

= Difference between stagnation pressure head (h_s) and static pressure (h_0)

If a differential manometer is connected to the tube of a Pitot static tube as in Fig. 5.8 it will measure the dynamic pressure head. The following figure shows the static pressure and stagnation pressure tube are combined into one instrument known as Pitot static tube. If *y* is the manometric difference, then

$\Delta h = y(\frac{\rho_m}{\rho} - 1)$

$\rho_m = density\ of\ manometeric\ liquid$

ρ = density of the liquid flowing through the pipe

$\therefore V = C\sqrt{2g\Delta h}$ or

$V = C\sqrt{2\left(\frac{\Delta p}{\rho}\right)}$

Where *Δp* is the difference between stagnation and static pressure. The value of *C* is usually determine from calibration test of the Pitot tube

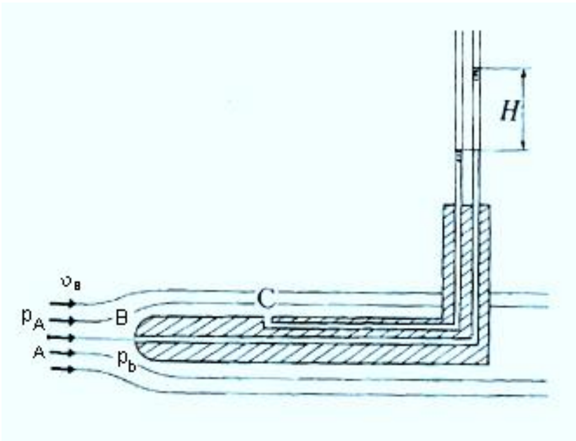


Figure 5.8: Pitot static tube.

Ex.1 A submarine fitted with a Pitot tube moves horizontally in sea. Its axis is ***12 m*** below the surface of water. The Pitot tube fixed in front of the submarine and along it axis is connected to the two limbs of a u-tube containing mercury, the reading of which is found to be ***200 mm***. Find the speed of the submarine.

Sol.

$\rho_{sea} = 1025\ kg/m^3$, $\rho_{mer} = 13600\ kg/m^3$

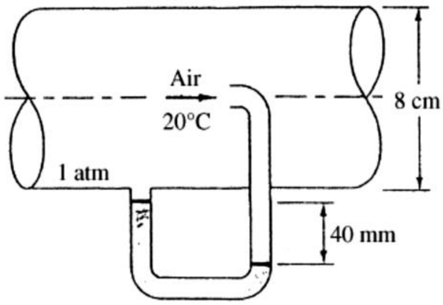
To find the head $\Delta h = y\left(\frac{\rho_m}{\rho} - 1\right) = 0.2(\frac{13600}{1025} - 1)$

$\Delta h = 2.45\ m$

$\therefore Velocity\ V = \sqrt{2g\Delta h} = \sqrt{2 * 9.81 * 2.45} = 6.94\ m/s$

$= 24.97\ km/hr$

Ex.2 For the Pitot-static pressure arrangement of the following figure, the manometer fluid is (colored) water at 20C°. Estimate (a) The centerline velocity, (b) The pipe volume flow rate, and (c) The smooth wall shear stress.



***Sol*.**

$\rho = 1.2 \frac{kg}{m^3}$, $\mu = 1.8 * 10^{-5} \frac{kg}{m.s}$ $for\ air.$

$\rho = 998 \frac{kg}{m^3}$, $\mu = 0.001 \frac{kg}{m.s}$ $for\ water.$

The manometer reads

$p_O - p = (\rho_{water} - \rho_{air})gh = (998 - 1.2)(9.81)(0.04)$

$p_O - p = 391\ Pa$

Therefore $V_{cl} = \left[\frac{2\Delta p}{\rho}\right]^{0.5} = \left[\frac{2(391)}{1.2}\right]^{0.5} = 25.5 \frac{m}{s}$

$Guess\ V_{av.} \approx 0.85\ V_{CL} \quad \approx 21.7 \frac{m}{s}$

$the\ volume\ flow\ rate\ is\ Q = \left(\frac{\pi}{4}\right)(0.08)^2\ (21.7) \approx 0.109 \frac{m^3}{s}$

$then\ \ Re = \frac{\rho VD}{\mu} = \frac{1.2(21.7)(0.08)}{1.8*10^{-5}}$

$Re = 115700$

$then\ f\ smoth\ \approx 0.0175$

$finally\ \tau_w = \frac{f}{8}\ \rho V^2 = \frac{0.0175}{8}(1.2)(21.7) \approx 1.23 Pas$

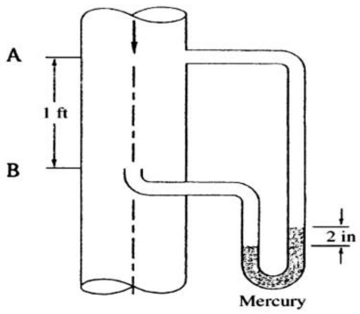
Ex.3

For the water flow of figure use the Pitot- static arrangement to estimate

a) The center line velocity
b) The volume flow in 5 in diameter smooth pipe
c) What error in flow rate is caused by neglecting the *(1 ft)* elevation difference?

Take:- $\rho = 1.94\ slug/\ ft^3$; $\mu = 2.09 * 10^{-5} \frac{slug}{ft.s}$

h = 2 in.



Sol.

For the manometer reading take the equal pressure at 0-0

$p_{OB} + (h + R)\rho_w g = p_A + h\,\rho_m g + R\rho_w g + 1 * \rho_w g$

$p_{OB} - p_A = h\rho_m g - h\rho_w g + \rho_w g = (\rho_m - \rho_w)hg + \rho_w g ----(a)$

Where R is the vertical distance between point B and the top level of mercury in right leg. From energy equation,

$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_{f-AB}$

$p_A - p_B = \rho g h_{f-AB} - \rho g(1\,ft)$ Static pressure difference ----------(b)

Therefor by summation Eq's (a) & (b)

$p_{OB} - p_A + p_A - p_B = (\rho_m - \rho_w)hg + \rho_w g + \rho g h_{f-AB} - \rho g$

$p_{oB} - P_B = (\rho_m - \rho_w)hg + \rho g\, h_{f-AB}$ Where h_{f-AB} friction losses.

$(p_{OB} - p_B) = (SG - 1)\rho g h = (13.56 - 1)(62.4)\left(\frac{2}{12}\right) \approx \frac{130.6 lb_f}{ft^2}$

$V_{CL} = \left(\frac{2\Delta p}{\rho}\right)^{0.5} = \left(2 * \frac{130.6}{1.94}\right)^{0.5} = 11.6\,\frac{ft}{s}$

$Q = AV_{CL} = \frac{\pi}{4}\left(\frac{5}{12}\right)^2 * 11.6 = 1.58\frac{ft^3}{s}$

$\Delta p_{friction} = \frac{f\left(\frac{l}{d}\right)\rho V^2}{2} \approx 3.2$ $3\%\ is\ neglecting$

$\Delta p\ pitot = 130.6 + 3.2 = 133.8\,psf$

Problems

P5.1 Oil flows through a ***160 mm*** pipe diameter with oil density ***950 kg/m³***. A venturimeter is fitted to the pipe line having ***110 mm*** throat diameter for measuring the flow rate of oil. The reading of mercury manometer is attached to it shows ***180 mm***. Determine the discharge of oil, assuming the coefficient of discharge for venturimeter as ***0.98***. ***[0.0725 m³/s]***

P5.2 Oil flows in ***0.18 mm*** pipe diameter with oil density is ***825 kg/m³***. A venturimeter ***0.1 m*** throat diameter is used to measure the flow rate of oil in pipe. The mercury manometer is attached to it showing a reading of ***0.2 m***, determine the coefficient of discharge of the venturimeter if the flow rate is ***0.055m³/s***. ***[0.85]***

P5.3 The throat diameter ***0.08 m*** of the venturimeter is used to measure the flow in the pipe line of ***0.15 m*** diameter. A mercury manometer attached to it shows deflection of ***0.3 m***. Assuming coefficient of discharge as ***1.0***. Calculate the flow rate in the pipe. ***[0.045 m^3/s]***

P5.4 A liquid flow rate ***6800 lit/min*** is measured by using venturimeter. The pressure difference across the venturimeter is equivalent to ***7.0 m*** of the flowing liquid. The pipe diameter is ***0.2 m***. Calculate the throat diameter of the venturimeter. Assuming the coefficient of discharge for the venturimeter as ***0.97***. ***[0.1096 m]***

P5.5 A ***0.075 m*** orifice diameter is fitted in an open tank under a head of ***4 m***. the actual velocity of liquid through the orifice is ***8 m/s***. If the flow rate measured in a collecting tank is ***0.022 m^3/s***. Determine the velocity coefficient, coefficient of contraction and the theoretical flow rate through the orifice. ***[0.9, 0.56, 0.62]***

P5.6 A pipe diameter is ***0.22 m*** carries oil with ***S.G=0.8***, an orifice of ***0.088 m*** diameter is fitted inside the pipe. The mercury manometer is attached across the orifice shows a reading of ***0.8 m***. Determine the actual flow rate through the pipe. Assume coefficient of discharge for the orifice as ***0.61***. ***[0.0587]***

P 5.7 A pitot static tube is used to measure the air velocity shows a reading of ***0.088 m*** through the water manometer is attached to pitot tube. Assuming the velocity coefficient is ***0.95*** and air density is ***1.22 kg/m^3***. Determine the air velocity. ***[35.71 m/s]***

P5.8 A mercury manometer attached to a pitot static tube is used to measure the water velocity in pipeline, it shows a reading of ***0.15 m***. Assuming the velocity coefficient is ***0.97***, determine the water velocity in pipe. ***[5.9 m/s]***

P5.9 The difference in stagnation and static pressure is ***0.09 m*** of water for a pitot static tube fitted in a pipe of ***0.25 m*** diameter. Assume the velocity coefficient as unity, calculate the water velocity in pipe line. ***[1.33 m/s]***

P5.10 The air velocity measured by pitot static tube is ***40 m/s***. The pressure difference recorded by the pitot static tube is ***0.125 m*** of water, if the density of flowing air is ***1.3 kg/m^3*** calculate the velocity coefficient. ***[0.92]***